

Tailoring Voltage Waveforms to Enhance Control of Gas Heating in Electrothermal Microthrusters

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Acknowledgements

Many thanks to James Dedrick, for providing excellent guidance as my supervisor during this project. Also thanks to Max Kellerman-Stunt for his invaluable support using HPEM. Thanks to authors of previous work on *Pocket Rocket* for their useful discussions, in particular Scott Doyle, Rod Boswell, Christine Charles and Amelia Greig.

Thanks to my parents and grandparents, for believing in me to embark on a masters of physics in the first place, and to everyone who I've met who has shaped my time at the University of York.

Abstract

Nanosatellites are a growing market for commercial and academic applications. However, mass and volume constraints prevent the implementation of conventional thrusters for orbit maintenance and collisions avoidance systems, limiting satellite lifetime. *Pocket Rocket*, a radio-frequency hollow cathode plasma microthruster, is being developed as a potential solution. This study investigates the use of tailored voltage waveforms to control neutral gas heating in *Pocket Rocket* under fixed operating conditions (150 V electrode potential, 325 K wall temperature, 100 sccm argon propellant flow, 0.85 Torr outlet pressure). The Hybrid Plasma Equipment Model is employed to perform 2D fluid/Monte-Carlo simulations varying θ_k in the peak-type tailored waveform. This results in an 86% modulation in the dc self bias. The largest dc self bias is produced using $\theta_k = 90^\circ$, resulting in γ -mode plasma heating. This is accessed at significantly lower voltage than the established standard sinusoidal voltage waveform. Varying θ_k provides a mechanism to dynamically alter gas heating properties electrically, allowing for thruster tuning when in orbit. Further comparison reveals a 5% increase in gas heating power efficiency using the $\theta_k = 90^\circ$ peak-type waveform compared to the sinusoidal. Control of gas heating mechanisms results in optimisation of exhaust gas velocity for use in nanosatellite applications.

Contents

1	Introduction	1
1.1	Motivation for <i>Pocket Rocket</i>	1
1.1.1	Nanosatellites and Their Contributions	1
1.1.2	Introduction to Rocket Propulsion	2
1.1.3	Introduction To Electric Propulsion	3
1.1.4	Introduction to <i>Pocket Rocket</i>	5
1.2	Plasma Physics of <i>Pocket Rocket</i>	6
1.2.1	Introduction to Plasmas	6
1.2.2	Introduction to Plasma Sheaths	7
1.2.3	Introduction to Capacitively Coupled Plasmas	8
1.2.4	CCP Asymmetry and the DC Self Bias Voltage	9
1.2.5	Electron Heating Mechanisms	10
1.2.6	Neutral Gas Heating in <i>Pocket Rocket</i>	11
1.2.7	Electrical Asymmetry Using Tailored Voltage Waveforms	12
1.3	State of the Art and Current Challenges	13
1.4	Scope of the Thesis	14
2	Method	15
2.1	Motivation for Hybrid Modelling of <i>Pocket Rocket</i>	15
2.2	Hybrid Plasma Equipment Model Simulation Geometry	16
2.3	Physics of Hybrid Plasma Equipment Model	17
2.3.1	Continuity Equations	17
2.3.2	Boltzmann Equation	18
2.3.3	Secondary Electrons	18
2.3.4	Flow Regimes	19
2.4	Iterative and Spatial Convergence	20
2.5	Simulation Conditions	21

2.6	Determining Plasma Parameters	22
2.6.1	DC Self Bias and Sheath Potential	22
2.6.2	Sheath Extent	23
2.6.3	Gas Heating Power Efficiency	24
2.7	Sources of Uncertainty	25
2.7.1	Temperature of Dielectric Tube Wall	25
2.7.2	Model Comparison with Fully Kinetic Treatment	25
3	Results and Discussion	27
3.1	<i>Pocket Rocket</i> Sinusoidal Waveform Operation	27
3.2	Tailored Waveform Modulation of DC Self Bias	30
3.3	Macroscopic Plasma Parameters in DC Self Bias Extrema of Peak-Type Waveform	31
3.4	Power Deposition Within the Plasma Sheath	34
3.5	Modulating Radial Ion Flux with DC Self Bias	36
3.6	Gas Heating Power Efficiency	38
3.7	Considerations for Future Work	39
4	Conclusion	41
5	Bibliography	43

1 Introduction

This chapter motivates the study of plasma micro-thrusters and introduces *Pocket Rocket*. A discussion on the plasma physics underpinning *Pocket Rocket* follows, before assessing the current challenges facing the thruster and this work’s contribution to its development.

1.1 Motivation for *Pocket Rocket*

1.1.1 Nanosatellites and Their Contributions

The first ever orbital satellite, *Sputnik*, was launched in 1957, weighing 84 kg and equipped with only a radio transmitter [1]. In the decades since, spacecraft have grown larger, heavier and more complex to allow for greater capabilities. However, the last twenty years has seen a renaissance for small, lightweight satellites, spurred by progress in miniaturization and the advent of launch platforms capable of carrying multiple payloads. Nanosatellites, such as CubeSats, have a mass between 1 and 10 kg and are typically on the scale of a 10 cm cube [2].

Since the conception of the CubeSat ‘standard’ for nanosatellites in 1999, nanosatellites have been used by space agencies, militaries, private companies, universities and even schools [3]. Their applications are varied: constellations of hundreds of nanosatellites have been deployed for telecommunications networks; Earth observation satellites are used to profile clouds and atmospheric chemistry; and deep-sky observing CubeSats employed as x-ray imaging telescopes [4]. Nanosatellite launch has been steadily increasing since the 2000s, and forecasts suggest that launch rates will continue at around 400 per year, as seen in Figure 1.1 [5].

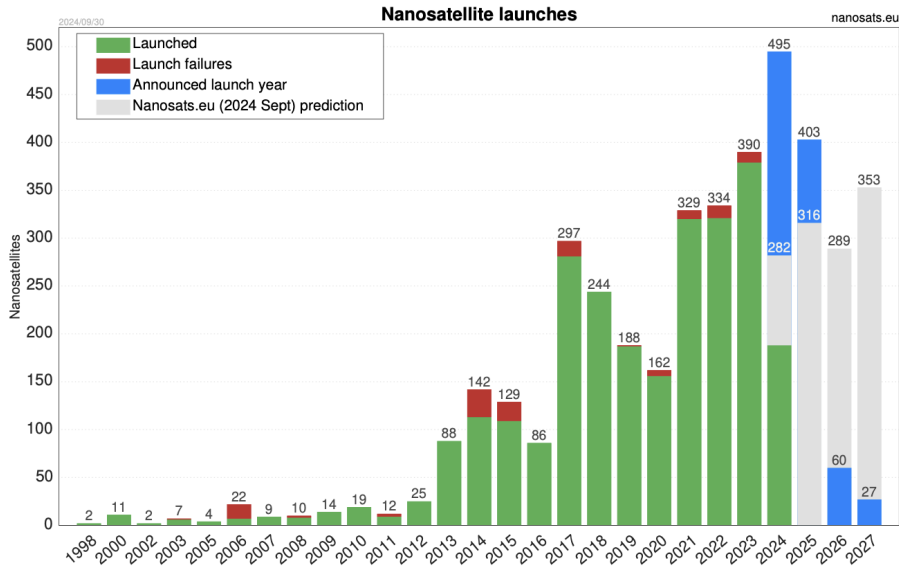


Figure 1.1: Nanosatellite launches from 1998 to 2024 with forecasted launches for 2025, 2026 and 2027 based on data from Nanosats Database. Figure reproduced from ref. [5].

These low volume spacecraft are too small for conventional propulsion systems to be implemented, therefore most nanosatellites are launched without onboard propulsion systems [6]. This limits a satellite's lifespan: most nanosatellites in low Earth orbit will de-orbit every 1-5 years [7]. An onboard propulsion system would enable the satellite to periodically re-install itself in orbit, counteracting atmospheric drag, and additionally used to avoid collisions with other spacecraft [8].

Nanosatellites are limited in their volume, mass and power – a CubeSat, for instance, is limited to a mass of 1.33 kg or less, including payload, propellant and thrust system [9]. Therefore, any thruster system needs to be lightweight as well as low volume. Furthermore, low power demand is essential, as nanosatellite solar panels typically generate only 10-20 W [2]. Thrust of approximately 1 mN is required for use as collision avoidance thrusters [8].

1.1.2 Introduction to Rocket Propulsion

The challenge of developing an effective propulsion system lies in maximising the effective thrust possible from a finite amount of fuel. Konstantin Tsiolkovsky was the first to consider this relationship in 1903 when he set out the rocket equation, given in Equation 1.1.1. [10].

$$\frac{m_f}{m_0} = \exp\left(\frac{\Delta v}{v_e}\right) \quad (1.1.1)$$

Here, m_f refers to the final mass of the rocket, m_0 its initial mass, Δv the change in velocity achieved and v_e the velocity of the exhaust gas. Δv is the objective parameter when designing a propulsion system; for a spacecraft to reach its intended destination its design must achieve a specific corresponding Δv .

The limiting case where mass-fraction $\frac{m_f}{m_0} \rightarrow 1$ gives the most efficient use of fuel. As such, to achieve a fixed Δv in the most efficient way, exhaust gas velocity, v_e , is maximised.

Thruster performance can also be considered in terms of the specific impulse, I_{sp} and thrust, F_T , given by Equation 1.1.2 and 1.1.3 respectively.

$$I_{sp} = \frac{v_e}{g} = \frac{I_m}{\dot{m}g} \quad (1.1.2)$$

$$F_T = v_e \dot{m} = g I_{sp} \dot{m} \quad (1.1.3)$$

Where g is the acceleration due to gravity at sea level on Earth and I_m is the impulse produced

by a propellant mass-flow rate $\dot{m}$. Specific impulse describes the length of time a propulsion system can accelerate its own initial mass at a rate of 1 g.

Crucially, mass fraction, specific impulse and thrust all have a positive dependence on exhaust velocity v_e , which can therefore be considered a crucial parameter to thruster performance.

The exhaust gas velocity is approximated as the mean velocity of the propellant. This is calculated by taking the first moment of the particle velocity distribution. High pressure gas flow through a thruster and out of the exhaust is a collisional system, therefore a Maxwellian velocity distribution can be assumed, shown in Equation 1.1.4 [11].

$$v_e \approx \langle v \rangle = \int_0^\infty v f(v) dv = \sqrt{\frac{8k_B T_g}{\pi m_g}} \quad (1.1.4)$$

Here, $f(v)$ is the particle velocity distribution function, k_B refers to the Boltzmann constant, T_g is the effective temperature of the gas and m_g is the mass of the propellant species. This shows that exhaust velocity, v_e , scales with $\sqrt{T_g}$. Therefore, a thruster's ability to increase the temperature of the propellant gas is key to maximising fuel efficiency for prolonged service and its performance producing specific impulse and thrust.

1.1.3 Introduction To Electric Propulsion

Electric propulsion offers a means of lightweight, low power thrust production for use in nanosatellites.

There are many variations on design, but broadly all electric propulsion systems can be classified as electrostatic, electromagnetic or electrothermal [12]. In each case, a plasma is formed when neutral gas undergoes ionisation, producing freely moving electrons and ions [13].

In electrostatic propulsion systems, such as gridded-ion thrusters, plasma ions are directly accelerated to high velocities by an electric field [14]. Similarly, electromagnetic systems accelerate ions via electric and magnetic fields. Electrothermal propulsion systems are distinct in that thrust is generated by heated neutral gas rather than the ions themselves.

A complete description of the operation of all electric propulsion systems is beyond the scope of this work, Table 1.1 instead provides an overview of operational parameters for a selection of technologies.

Table 1.1: A summary of operational parameters for a selection of electric propulsion technologies. Values reproduced from ref. [15].

Propulsion Type	Technology	Thrust (mN)	Specific Impulse (s)	Power (W)
Electrostatic	Gridded-Ion	0.1 - 1.5	≈ 3000	20 - 60
Electromagnetic	Hall-Effect	0.1 - 16	830 - 1718	50 - 300
	Mini-Helicon	1 - 4	600 - 1200	50 - 300
Electrothermal	Arcjet	0.014 - 0.031	313	119
	Resistojet	109-335	93 - 385	167 - 506
	Radio-frequency hollow cathode thruster	1.13 - 2.0	30 - 70	1.8 - 4.8

From this comparison, the radio frequency (rf) hollow cathode thruster is presented as the best candidate for nanosatellite applications. It is unique in its low power draw (< 10 W) while producing sufficient thrust (> 1 mN) to be used in collision avoidance systems. Furthermore, being an electrothermal system, it is less complex and easier to miniaturise for applications with nanosatellites [16]. This is primarily due to not requiring a neutraliser: in electrostatic and electromagnetic systems, accelerated ions are neutralised to prevent charge build up on the spacecraft [17]. Neutralisers require additional power, are difficult to scale-down and are particularly fragile components during satellite launch [18].

The next section introduces a practical implementation of the rf hollow cathode thruster studied in this work.

1.1.4 Introduction to *Pocket Rocket*

Pocket Rocket is a radio-frequency hollow cathode plasma thruster, under development at the York Plasma Institute at the University of York in collaboration with the Australian National University and the Space Plasma, Power and Propulsion Laboratory [19]. Figure 1.2 shows *Pocket Rocket* operating inside a CubeSat frame.

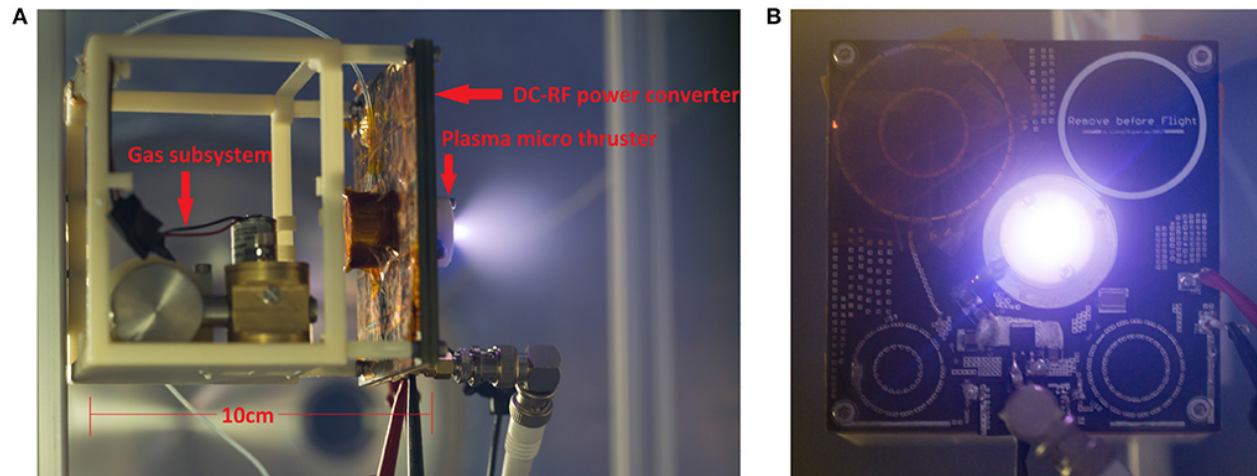


Figure 1.2: Profile and face-on image of *Pocket Rocket* in operation inside CubeSat frame. Figure reproduced from ref. [20].

Pocket Rocket has been established as a viable means of generating thrust for microsatellites, producing milli-Newtons of thrust while drawing less than 10 W in power [21]. *Pocket Rocket* has been demonstrated to fit within a 1U Cubesat, and a prototype miniature version has successfully produced a plasma while in a vacuum [22]. The next section discusses *Pocket Rocket*'s operation and mechanism of generating thrust.

1.2 Plasma Physics of *Pocket Rocket*

Pocket Rocket's operation is based around a physically asymmetric capacitively coupled plasma. The following sections outline how a capacitively coupled plasma is formed and can be controlled using electrical asymmetry.

1.2.1 Introduction to Plasmas

A plasma is formed when a neutral gas is ionised, producing freely moving electrons and ions. Ionisation occurs when a free electron in the neutral gas is accelerated by an electric field and collides with neutral atoms. If the electron is accelerated to an energy higher than the neutral atom's ionisation potential, the collision will result in the liberation of a further electron and production of an ion. The liberated electron is accelerated by the electric field and the process repeats. The fraction of species that become ions is quantified in the degree of ionisation, α , given in Equation 1.2.1. In *Pocket Rocket*, a weakly ionised plasma is produced, with $\alpha \ll 1$.

$$\alpha = \frac{n_i}{n_i + n_n} \quad (1.2.1)$$

The bulk of a plasma exists in quasi-neutrality, where the number of ions is approximately equal to that of electrons. Both species move freely, with the electrons moving in response to an electric field, thus the plasma acts as a conductor [13].

1.2.2 Introduction to Plasma Sheaths

An idealised plasma existing in free space exists entirely in quasi-neutrality. However, practical plasmas are confined within walls of a volume. Near these boundaries, plasma behaves differently; between the bulk and the wall we see areas of net positive charge. These regions are known as the plasma sheath. Figure 1.3 shows the ion and electron densities in a plasma close to the wall of a container.

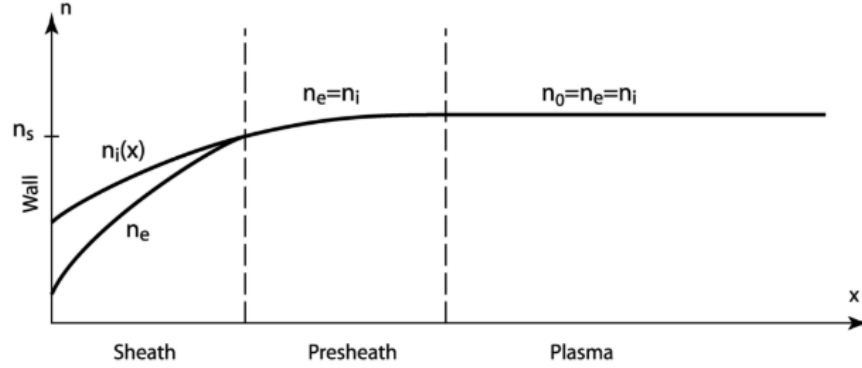


Figure 1.3: Sketch of ion and electron densities, n_i and n_e , respectively, close to the wall of a plasma container. Sheath region forms adjacent to wall ($n_i > n_e$) and presheath region ($n_i \approx n_e$) between wall and plasma bulk ($n_0 \approx n_i \approx n_e$). Figure reproduced from ref. [13].

When the neutral gas initially breaks down into plasma, free electrons move with velocity approximately a hundred times faster than ions as they are lower mass [23]. Their movement is unconfined and therefore the electrons are quickly absorbed by the walls. This results in the formation of an area of overall positive charge close to the wall ($n_i > n_e$). A pre-sheath separates the bulk from the sheath, a full consideration of this region is beyond the scope of this work.

1.2.3 Introduction to Capacitively Coupled Plasmas

In this work, power is capacitively coupled into a plasma. A typical radio frequency capacitively coupled plasma (CCP) is shown in Figure 1.4.

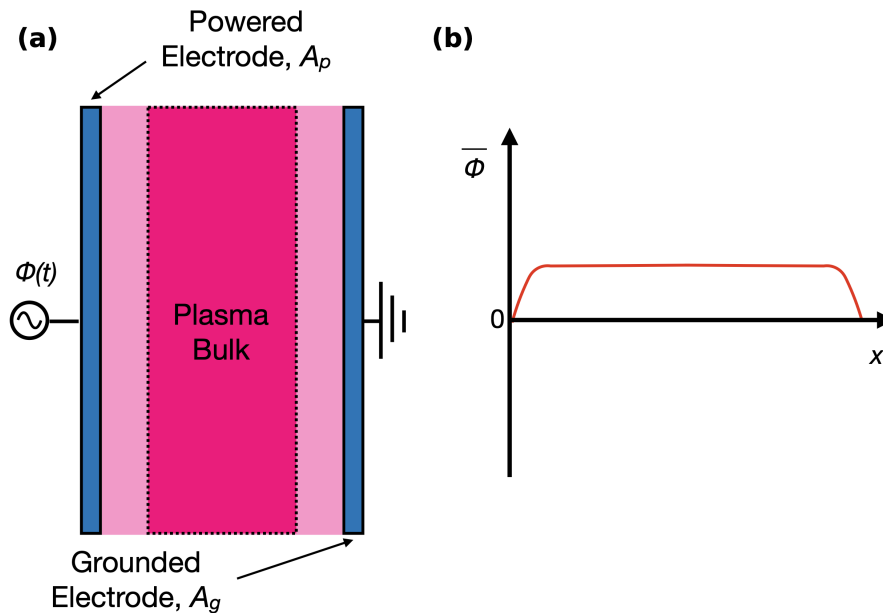


Figure 1.4: (a) Diagram of a physically symmetrical capacitively coupled plasma, with two electrodes (blue) of equal surface area. The powered electrode is driven by a radio-frequency voltage, generating a potential $\phi(t)$. Between them, a plasma forms with the bulk in the centre (dark pink) and two sheath regions adjacent to either electrode (light pink). (b) Sketch of the time-averaged plasma potential, $\bar{\phi}$, between the two plates, region of decrease on either side represents position of the plasma sheath.

Figure 1.4(a) shows a powered and grounded electrode of equal surface area, either side a region of neutral gas, separated by a distance in x . The electrodes are each separated from the gas by a layer of dielectric [13]. A rf voltage is applied to the powered electrode and an electric field forms, causing free electrons in the gas to accelerate to collide with neutral atoms. This results in the liberation of further electrons from ions and the neutral gas breaks down into a plasma. Typical driving voltages are of the order 100-1000 V, gas pressure is typically 10-100 mTorr and the standard electrode voltage frequency is 13.56 MHz [23]. The resultant, phase-averaged potential between the two electrodes is shown in Figure 1.4(b). Note the potential is symmetrical across x , with the regions of potential decrease on either side the positions of the plasma sheaths.

1.2.4 CCP Asymmetry and the DC Self Bias Voltage

Pocket Rocket employs an additional modification to the CCP scheme by introducing a physical asymmetry. An illustration of a simple asymmetric CCP is given in Figure 1.5(a), with the grounded electrode larger than the powered ($A_g > A_p$). Figure 1.5(b) depicts the resultant phase averaged potential between the electrodes.

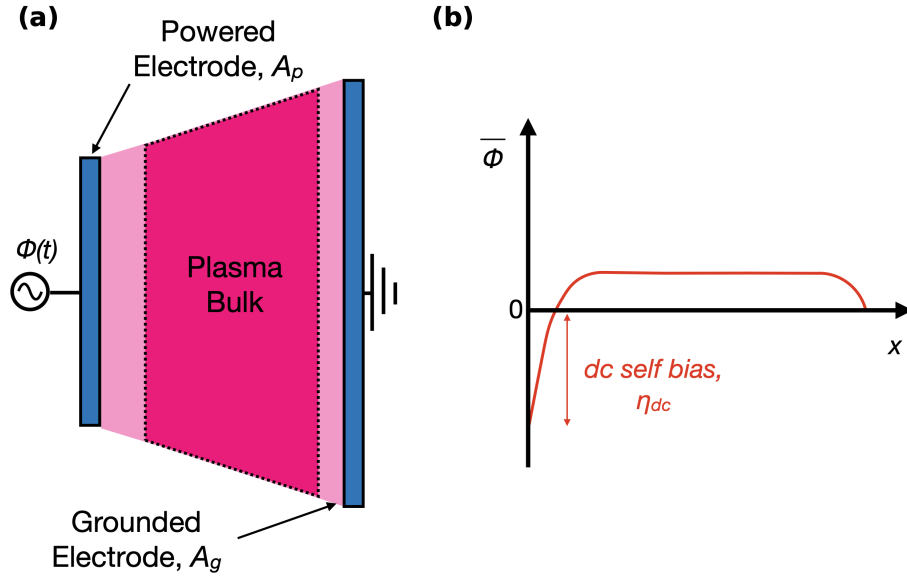


Figure 1.5: (a) Sketch of a physically asymmetrical CCP with the ground electrode area larger than the powered ($A_g > A_p$). Sheath regions are indicated in light pink, the plasma bulk in dark pink and electrode surfaces in blue. (b) Sketch of the time-averaged plasma potential, $\bar{\phi}$, within a physically asymmetrical CCP. A negative potential forms at the powered electrode, the dc self bias, in order to make the current density through the powered electrode greater than the grounded and conserve charge in the system.

In an asymmetric CCP, the sheath regions can be modelled as capacitors [23]. For a plasma to maintain quasi-neutrality, charge must be conserved within the system; the total charge entering and leaving must be equal over one rf-cycle [24]. For this to be true, the current density through the powered electrode, with smaller surface area, must be greater than that through the grounded. This means an accelerating potential is formed at the powered electrode in order to maintain the conservation of charge in the system. This potential is called the dc self-bias voltage and can be considered to represent the difference between the voltage at the powered and grounded electrodes, shown in Figure 1.5(b). The dc self-bias voltage, η_{dc} , varies with the ratio of electrode surface areas and the ratio of sheath extents, as shown in Equation 1.2.2 [23].

$$\eta_{dc} \propto \left(\frac{A_g}{A_p}\right)^k \left(\frac{\hat{S}_p}{\hat{S}_g}\right) \quad (1.2.2)$$

Where $\hat{S}_p$ and $\hat{S}_g$ are the phased average powered and grounded sheath extent, respectively.

The value k is generally found to be a value between 2 and 3 depending on the geometry of the reactor and the electrode properties [23]. To a first order approximation, applicable for this consideration of *Pocket Rocket*, the ratio of the sheath extents is 1. In this case, maximising the ratio of ground to powered electrode will maximise the generated dc self bias voltage.

1.2.5 Electron Heating Mechanisms

Power is coupled into a plasma by heating electrons, as these are they have high mobility to react to temporal variations in the applied voltage. In comparison, the plasma ions have larger mass and so reduced mobility, therefore only reacting to time-averaged effects.

Electron heating occurs in rf CCPs in two distinct regimes, α -mode and γ -mode. α -mode can be considered as follows: over an rf-cycle, the sheath edge in the plasma is moving, contracting and expanding at points of maximum $\frac{dV}{dt}$. As the sheath expands, it will collide with electrons, causing the electron to gain momentum, as in a classical model of a rigid ball reflected from a moving wall [13]. α -mode is the typical mechanism of plasma heating seen at low voltages in rf CCPs.

γ -mode heating typically is generated when operating using higher voltages. This results in significant ion bombardment on the wall surfaces, leading to increased secondary electron production via the Auger process [25]. These secondary electrons are then accelerated electrostatically through the sheath potential in to the plasma, allowing for greater power deposition compared to collisional electrons. Secondary electrons can then undergo multiple ionisation in the bulk to produce further ions to increase wall bombardment, leading to an avalanche effect.

1.2.6 Neutral Gas Heating in *Pocket Rocket*

Pocket Rocket has a cylindrical architecture with gas flow surrounded by concentric annulus-shaped electrodes and dielectric layer. This is illustrated in a simplified sketch of a cross-section of the thruster in Figure 1.6.

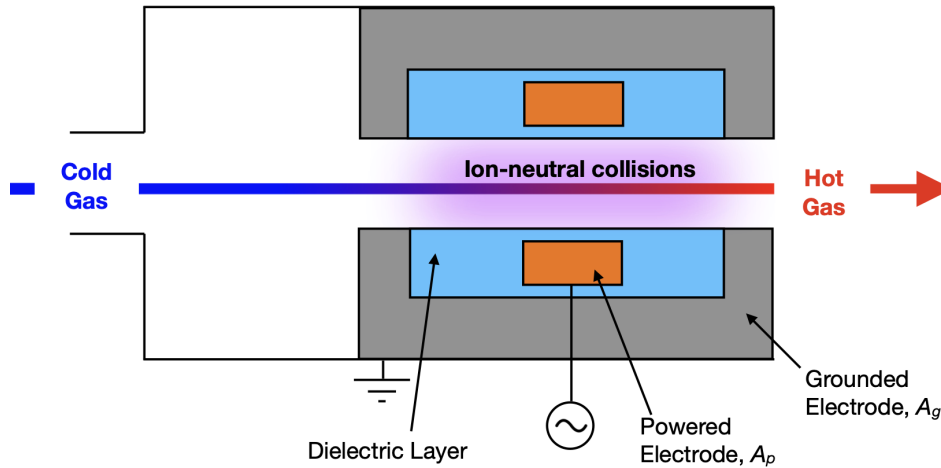


Figure 1.6: Sketch of cross-section of *Pocket Rocket* hollow cathode thruster. Cold gas propellant enters the thruster plenum, a radio-frequency voltage is applied to the powered electrode (orange) and power coupled to gas through dielectric tube (light blue). The powered electrode has surface area smaller than the grounded electrode ($A_p < A_g$), leading to formation of a physically asymmetric CCP. Gas is partially ionised and ion-neutral collisions occur resulting in neutral gas temperature increase. Hot, neutral gas is expelled from the exhaust.

Argon gas propellant enters the thruster plenum before entering the dielectric tube. A radio-frequency voltage is applied to a powered electrode, adjacent to the plasma and separated by a dielectric layer. The powered electrode has a surface area much smaller than the grounded electrode surrounding it, therefore generating a physically asymmetric CCP. The plasma then volumetrically heats the propellant primarily via ion-neutral charge exchange collisions [19, 26, 27]. For argon gas used in *Pocket Rocket*, this reaction is described by Equation 1.2.3.



Therefore, altering ion production by changing the electron heating mechanism will affect the neutral gas heating properties. Secondary electrons produced in γ -mode have very large energies (≈ 50 eV) [28] compared to the ionisation energy of the argon gas (≈ 14 eV) [29]. Therefore, they undergo multiple ionisation producing significantly more ions in the plasma bulk than α -mode, stimulating increased ion-neutral collisions and gas heating [13].

1.2.7 Electrical Asymmetry Using Tailored Voltage Waveforms

The dc self-bias effect has been generated by considering a physical asymmetry between the electrodes in a CCP. Industrial demands of CCPs led to investigation as to whether the same effect can be achieved electrically in a fixed physical system to allow more control over plasma behaviour [30]. Initial studies using distinct high- and low-frequency waveforms did not produce the independent control of the plasma properties demanded [31, 32], prompting the development of ‘tailored’ voltage waveforms [33].

Tailored waveforms sum a fundamental frequency with a set number of harmonics with a phase offset between each. This has been demonstrated to allow independent control of plasma density and electron and ion energies, known as the electrical asymmetry effect (EAE) [34, 35].

One such tailored waveform that has demonstrated significant independent control of plasma parameters is the ‘peak-type’ waveform, presented in Equation 1.2.4 [36].

$$\phi_{rf}(t) = \sum_{k=0}^n \left(\frac{\phi_0}{n} \right) \sin(k\omega_0 t + \theta_k) \quad (1.2.4)$$

Where $\phi_{rf}(t)$ gives the resultant, combined voltage waveform, ϕ_0 the maximum amplitude, ω_0 the fundamental angular frequency and θ_k the phase offset of harmonic k for n total harmonics ($n = 5$ in this work). The effects of varying θ_k in the peak-type waveform are investigated in this study. Figure 1.7 compares the $\theta_k = 0^\circ$, 90° and 270° tailored waveforms with a sine wave for clarity.

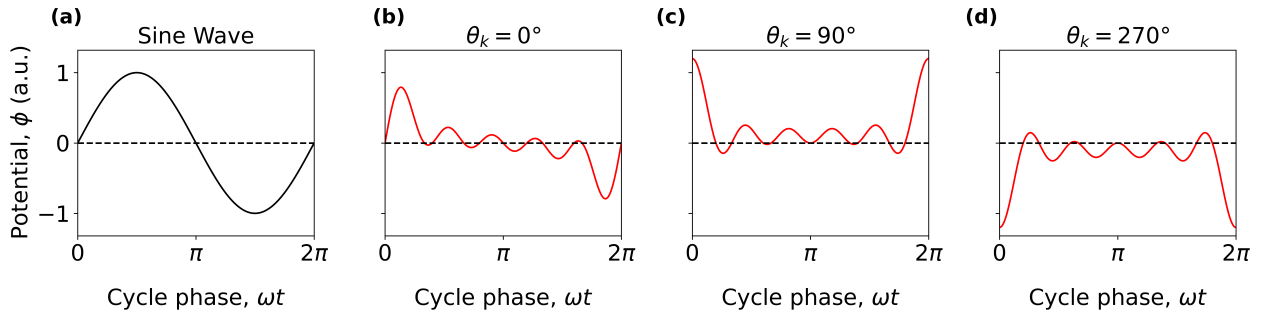


Figure 1.7: Comparison of (a) sinusoidal waveform with ‘peak-type’ tailored waveforms, as described in Equation 1.2.4, using $\theta_k =$ (b) 0° , (c) 90° and (d) 270° .

1.3 State of the Art and Current Challenges

Pocket Rocket has been in development for over a decade [19]. Initial work in ref. [21] confirmed *Pocket Rocket*'s viability as a microthruster through investigation of the neutral gas heating mechanisms via ion-neutral collisions and wall heating. Further work in ref. [37] simulated *Pocket Rocket* using different cold gases, modelling flow and thrust behaviour to compare experimental result with simulation. During this period, work was completed in ref. [38] to refine the physical thruster geometry to maximise RF power coupling at a fixed voltage.

Later research has focussed on developing the electrical methods of controlling and optimising *Pocket Rocket*. Electrical manipulation enables a method of dynamically altering *Pocket Rocket*'s thrust characteristics remotely when installed in an operating spacecraft. Typically *Pocket Rocket* is operated using a sinusoidal 13.56 MHz voltage waveform. Refs. [15, 39] showed that varying the electrode voltage amplitude can significantly change electron and ion heating dynamics; at high-voltages, plasma heating shifts from α -mode to γ -mode as a result of increased secondary electron emission. Ref. [40] investigated varying the frequency of the sinusoidal waveform at a fixed voltage to establish that low voltage frequencies increase gas temperature more than high frequencies per unit of deposited power. This is due to γ -mode being the primary electron heating mechanism at low frequencies.

A strong line of inquiry now is to investigate applying tailored voltage waveforms to *Pocket Rocket*. Ref. [41] demonstrated that dual-frequency waveforms enable greater control over plasma power deposition and gas temperature. Further work demonstrated that phase-offset of harmonics can modulate the dc self-bias voltage without needing to change the powered electrode voltage amplitude [15]. A natural extension is to investigate more complex waveforms with a larger number of harmonics. Ref. [15] demonstrated that a 'sawtooth' waveform with variable phase-offset could be employed as a control to modulate the dc self-bias voltage by 15%. Further study in ref. [42] introduced the use of peak-type waveforms, enabling dc self bias modulations of up to 85%, and with it greater independent control of the ion flux and mean ion energy compared to dual-frequency waveforms.

However, no study of the gas heating capability of the peak-type waveform has yet been undertaken, crucial to assessing its feasibility for use in a thruster. As such, this work investigates how varying the phase offset, θ_k , of the peak-type waveform affects neutral gas heating in *Pocket Rocket*.

1.4 Scope of the Thesis

The remainder of this dissertation is organised as follows:

- **Chapter 2** introduces the methodology of the project, starting with the Hybrid Plasma Equipment Model, its usage with *Pocket Rocket*, and then the key physics that underpins its usage.
- **Chapter 3** discusses the results of simulating *Pocket Rocket* under a sinusoidal waveform regime, before comparing to operation using the peak-type tailored waveform. The resulting change in plasma behaviour due to phase offset variation is established and its effect on gas heating explored.
- **Chapter 4** summarises and concludes the results of this work.

2 Method

This chapter introduces the Hybrid Equipment Plasma Model used to simulate *Pocket Rocket* and compares it to alternative modelling approaches. A discussion of its underpinning physics and application to *Pocket Rocket* follows. Methods to determine the dc self bias, sheath potential, sheath extent and gas heating power efficiency are considered before summarising the primary sources of uncertainty in this study.

2.1 Motivation for Hybrid Modelling of *Pocket Rocket*

Pocket Rocket's small scale, with an internal diameter of 2.1 mm, makes diagnostics practically difficult [19]. To develop an understanding of plasma behaviour within *Pocket Rocket*, simulations are employed. However, accurately modelling a real-world plasma is challenging. There are three main methods of simulation: kinetic models, fluid models and hybrid models.

Kinetic models, such as particle-in-cell simulations (PIC) simulate the motion of macro-particles in a plasma by solving the equations of motion for each over a series of time steps [43]. These equations are solely dictated by the electromagnetic and mechanical forces within the simulation volume. PIC models are considered the 'gold standard' in plasma physics, as they are entirely self-consistent and make no assumptions about internal electric fields or particle velocity distributions [44]. However, this makes them computationally expensive, and previous PIC models of complex geometries and chemistry sets, such as those in *Pocket Rocket*, have required impractical computational resources and timescales for the scope of this work [45].

Fluid models save computational expense by treating the plasma macroscopically and applying fluid equations to describe the overall motion. This assumes a Maxwellian distribution of velocities for all particles. This is less computationally demanding, however thus assumes only collisional energy transfer [46]. As a result, fluid models cannot be used to reliably simulate lighter species, such as secondary electrons emitted from a material surface [47]. Furthermore, at low pressures (< 10 Pa), collisionless interactions dominate, which a fluid model is unable to capture [48].

A hybrid model combines a kinetic and fluid model to produce a compromise, considering the demands of the simulation geometry, boundary conditions and computational expense. The Hybrid Plasma Equipment Model (HPEM) combines a 2D fluid and Monte-Carlo collision model [49]. The fluid model describes macroscopic plasma behaviour, while the Monte-Carlo model accounts for kinetic interactions of secondary electrons in the plasma. HPEM splits a simulation between its specialised modules for specific physical processes, enabling more efficient computing while maintaining accuracy. The modularity of the model also makes it robust and applicable to a range of plasma systems.

2.2 Hybrid Plasma Equipment Model Simulation Geometry

HPEM simulates a 2D ‘mesh’ that represents a cross-section of a cylindrically symmetrical system. The simulation area described by the *Pocket Rocket* mesh is shown in Figure 2.1, using a 64×152 grid with mesh-point resolution $R \times Z = 0.125 \times 0.5$ mm. The radius of the thruster is simulated as 4 mm and the axial length 76 mm. This mesh has been used in previous work benchmarked against experiment with *Pocket Rocket* [39, 41, 42].

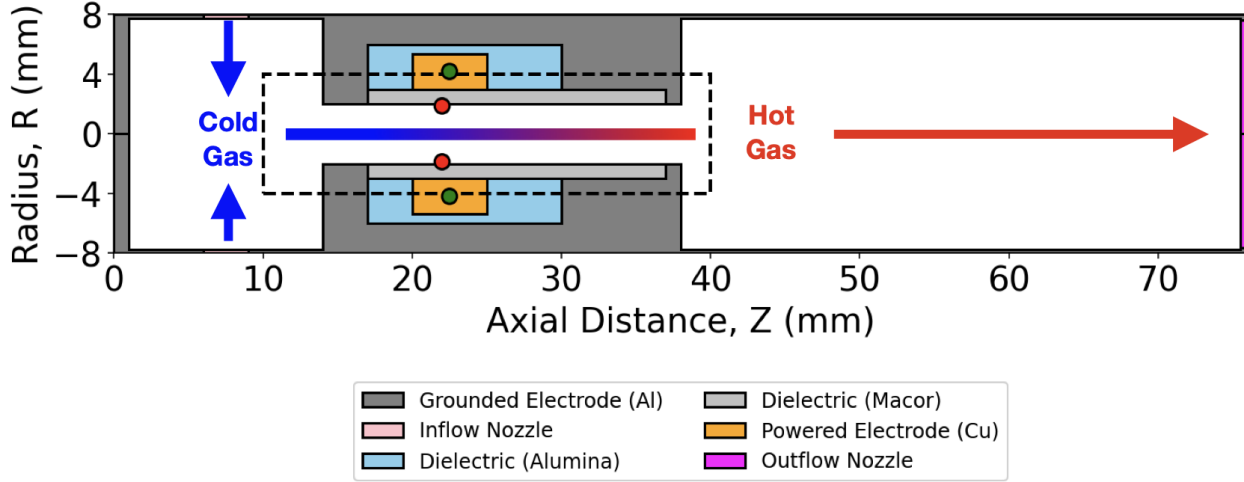


Figure 2.1: Schematic of the *Pocket Rocket* mesh used in HPEM simulations, represents a 2D radial slice of a cylindrically symmetrical system of radius 4mm and axis length 76mm (diagram reflected to show full diameter of thruster). The powered copper electrode (orange) is surrounded by dielectric (blue, silver) within a grounded aluminium wall housing (grey). argon gas inflows through the inlet nozzle (pink) at a fixed rate of 100 sccm and outflows at a fixed pressure of 0.85 Torr (magenta). The dashed rectangle indicates the ‘throat’ of the thruster, $R = -4$ to $+4$ mm and $Z = 10$ to 40 mm. The green dot indicates the position the powered electrode potential was measured and the red dot the position of the dielectric surface potential, used to calculate the dc self bias.

2.3 Physics of Hybrid Plasma Equipment Model

2.3.1 Continuity Equations

For a plenum pressure of 1.4-1.7 Torr, ions, neutral species and bulk plasma electrons in the plasma are assumed to have a Maxwellian energy distribution due to collisional processes dominating. This means that they can be modelled as a fluid. Previous work ascertained this model was valid based on comparing simulations and real *Pocket Rocket* experimental data [39, 41].

A fluid model describes particle behaviour based on continuity equations for mass, momentum and energy. For the heavy particles and electrons, the mass continuity equation is given by Equation 2.3.1.

$$\frac{\delta n_s}{\delta t} = -\nabla \cdot \Gamma_s + S_s \quad (2.3.1)$$

Here, n_s refers to the number density of species s , Γ_s gives the flux of particles between adjacent cells and S_s is the particle's source function. For the bulk electrons, the electric field is weak due to quasineutrality and collisional interactions dominate, therefore the drift-diffusion approximation is applied to determine the bulk electron flux Γ_e . This is given in Equation 2.3.2.

$$\Gamma_e = -n_e \mu_e E - D_e \nabla n_e \quad (2.3.2)$$

Here, E refers to the electric field, μ_e refers to the electron mobility coefficient and D_e to the electron diffusion coefficient. These coefficients are discussed further at the end of this section. The momentum continuity equation for the ions and neutral particles is given by Equation 2.3.3.

$$\frac{\delta \Gamma_i}{\delta t} = \underbrace{-\frac{1}{m_i} \nabla(n_i k_b T_i)}_{\text{Thermal motion}} - \underbrace{\nabla \cdot (n_i v_i^2)}_{\text{Self-collisions}} + \underbrace{\frac{q_i}{m_i} n_i E}_{\text{Lorentz force}} - \underbrace{\nabla \cdot \bar{\bar{v}}_i}_{\text{Viscous forces}} - \underbrace{\sum \frac{m_j}{m_i m_j} n_i n_j v_{ij} (v_i - v_j)}_{\text{Elastic collisions}} \quad (2.3.3)$$

Here, T_i , n_i and m_i refer to the mean temperature, number density and mass of species i respectively; $\bar{\bar{v}}_i$ is the viscosity tensor and v_{ij} is the collision frequency between species i and j . Neutral species exclude the Lorentz force term due to being unaffected by the electric field, and ions exclude viscosity terms due to low density compared to neutral species.

Bulk electron energy conservation is described by Equation 2.3.4.

$$\frac{\delta \Gamma_e}{\delta t} = -\nabla \Gamma_e - e \Gamma_e E \sum_j n_e n_j k_{L,ej} \sum_j 3 \frac{m_e}{m_j} k_{ej} k_b n_e (T_e - T_j) \quad (2.3.4)$$

Here, n_j , m_j , T_j refer to the number density, mass and temperature respectively for the species j for which electron impact collisions are specified. k_{ej} and $k_{L,ej}$ give the elastic and inelastic energy loss rate coefficients respectively between electrons and species j and are obtained from

the mean electron temperature $\hat{T}_e$.

For the heavy particles, the energy continuity equation is extended to Equation 2.3.5.

$$\begin{aligned}
 \frac{\delta N_i c_i T_i}{\delta t} = & \underbrace{\nabla \cdot \kappa_i \nabla T_i}_{\text{Heat conduction}} - \underbrace{P_i \nabla \cdot \mathbf{v}_i}_{\text{PdV work}} - \underbrace{\nabla \cdot (n_i \mathbf{v}_i \epsilon_i)}_{\text{Heat convection}} + \underbrace{\frac{n_i q_i^2}{m_i v_i} \mathbf{E}^2}_{\text{Electrostatic potential}} + \underbrace{\sum_j \frac{m_j}{m_i m_j} n_i n_j v_{ij} k_B (T_j - T_i)}_{\text{Elastic collisions}} \\
 & \pm \underbrace{\sum_j n_i n_j k \Delta H_{ij}}_{\text{Change in enthalpy}} \pm \underbrace{\sum_j 3 n_i n_j k_{ij} k_B T_j}_{\text{Inelastic collisions}}.
 \end{aligned} \tag{2.3.5}$$

Here, N_i is the number density of the species, c_i is the specific heat capacity and T_i the species temperature, so the left-hand side, $\frac{\delta}{\delta t}(N_i c_i T_i)$, gives us the time evolution of the internal energy of the species i . κ_i gives the species conductivity, P_i the partial pressure and ϵ_i is the specific energy ($\epsilon_i = c_i T_i$). ΔH gives the change in enthalpy for the reaction and $k_{\Delta H_{ij}}$ the corresponding reaction rate coefficient.

2.3.2 Boltzmann Equation

To attain the electron mobility and electron diffusion coefficients necessary to solve Equation 2.3.2, HPEM calculates the electron energy distribution function and mean electron temperature, $\hat{T}_e$ via a two-term approximation of the Boltzmann equation [50]. These solutions use a series of manually specified reduced electric fields and electron impact cross-sections. μ_e and D_e are resultantly calculated using Equations 2.3.6 and 2.3.7 respectively. Here, v_{drift} is the scalar value of electron drift velocity and E the scalar value of electric field.

$$\mu_e = \frac{v_{\text{drift}}}{eE} \tag{2.3.6}$$

$$D_e = \frac{k_b \hat{T}_e \mu_e}{e} \tag{2.3.7}$$

2.3.3 Secondary Electrons

Secondary electron emission is modelled separately from the bulk electron and heavy species as the same assumptions for a Maxwellian energy distribution cannot be applied [39]. Instead, secondary electrons are tracked kinetically via the Monte Carlo method.

Secondary electrons are emitted from the thruster walls within the simulation at a proportional rate to ion flux incident to the wall [51]. This secondary electron coefficient, γ , is defined to be $\gamma = 0.2$ at the surface of the alumina dielectric and $\gamma = 0$ otherwise, as used in previous studies of *Pocket Rocket* [39]. This means that secondary electron production only occurs in the ‘throat’ region, and one secondary electron is produced for every five incident ions to the

alumina surface. Secondary electrons produced are all launched with an energy of 3 eV before being accelerated through the local potential gradient. From here the Monte Carlo algorithm tracks the evolution of the electrons without any additional assumptions about their behaviour.

2.3.4 Flow Regimes

For this model of *Pocket Rocket*, argon gas enters the thruster at a fixed inlet flow rate and exits at a fixed outlet pressure. The neutral flow between mesh cells is then calculated from the pressure gradient that arises between these two boundary conditions [15].

When the flow enters the ‘throat’, where the volume the gas occupies decreases, momentum-flux of gas particles will increase. As such, collision rates will change. ‘Collisionality’ can be described by the Knusden number, K_n , the ratio between atomic mean free path, λ and a characteristic dimension of the restricting geometry, L [52]. In this case, L is proportional to the radius of the throat region, on the order of 2 mm.

$$K_n = \frac{\lambda}{L} \quad (2.3.8)$$

In this work, *Pocket Rocket* is simulated in conditions that give a resulting Knusden number of the order $0.01 \leq K_n \leq 0.35$ [15]. As a result, the thruster operates at the boundary between a slip-flow and transitional-flow regime. This means that collisional effects are significant enough that fluid velocity of the gas parallel to the thruster wall can’t be assumed to be zero, but not so significant that a full kinetic treatment of the particles is required. Therefore a ‘slip-flow’ condition is applied to maintain non-zero velocities at all plasma-material interfaces in the thruster [53].

2.4 Iterative and Spatial Convergence

Two forms of convergence are considered in simulating *Pocket Rocket*: iterative convergence, where an individual simulation approaches a steady-state solution with increasing iterations, and spatial convergence, where an iteratively converged simulation is invariant compared to the mesh resolution.

Iterative convergence was established for each simulation by comparing the mesh-averaged value of electron density, bulk electron source rate, plasma potential and electron temperature. If the variation was less than 5% by the final iteration the simulation was considered converged. Figure 2.2 gives an example temporal convergence plot for the peak-type tailored waveform operation under baseline operating conditions and $\theta_k = 0^\circ$.

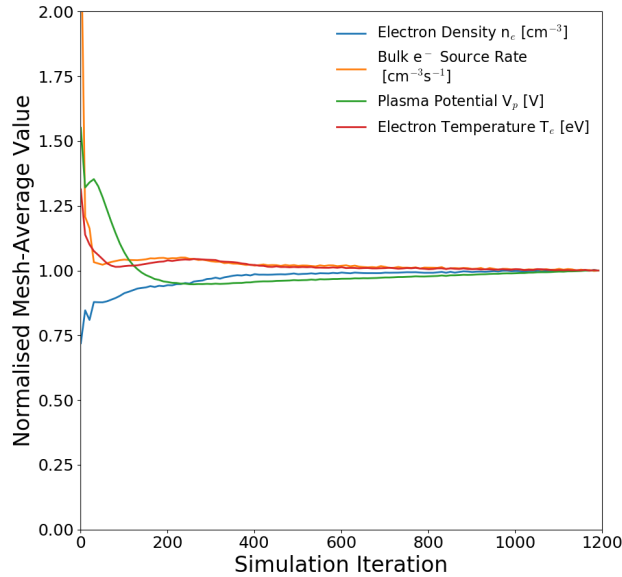


Figure 2.2: Temporal convergence of *Pocket Rocket* simulation using peak-type tailored waveform with $\theta_k = 0^\circ$, under baseline operating conditions given in Table 2.1. Convergence was achieved when the variation in the normalised mesh values of electron density, bulk electron source rate, plasma potential and electron temperature were within 5%.

For each phase-offset peak-type waveform simulation, convergence was achieved when using 1200 iterations. For the sinusoidal simulation under the same conditions, only 500 iterations was required.

Previous work established the *Pocket Rocket* mesh described in Figure 2.1, containing 9728 cells, is the smallest mesh to achieve spatial convergence under baseline operating conditions [15]. While increasing the mesh resolution would increase simulation reliability, it would also increase computational expense, and the chosen mesh represents the optimum trade-off of the two.

2.5 Simulation Conditions

All simulations in this study were performed using a powered electrode voltage of 150 V. This voltage was chosen as previous work demonstrated it could sustain a plasma inside *Pocket Rocket* under α -mode operation [39]. Each simulation took approximately one week to converge, and higher operating voltages increased this period, therefore they weren't investigated due to time constraints.

The fundamental frequency of the sinusoidal and peak-type rf voltage waveform in each case was 13.56 MHz. This frequency was chosen as it has been established as a standard in previous work using *Pocket Rocket* for sinusoidal and peak-type waveform operation and would allow for comparison with the literature [19, 39, 42].

For each simulation, the wall temperature in *Pocket Rocket* was fixed at 325 K. This temperature was chosen as previous work demonstrated that HPEM models of *Pocket Rocket* at 325 K produce qualitatively similar on-axis gas temperature variations when compared to different computational models [15, 27, 28].

Pocket Rocket was simulated with an argon gas propellant input flow rate of 100 sccm at the gas inlet and a fixed outlet pressure of 0.85 Torr. These values were selected to align with the literature as previous work has demonstrated these restrictions to produce plenum pressures closely matching those produced in experimental setups [15, 39, 41].

These operating conditions are summarised in Table 2.1 and henceforth referred to as baseline operating conditions throughout this work.

Table 2.1: Summary of baseline operating conditions used with *Pocket Rocket* in this work to allow comparison with previous results in the literature.

Operating Parameter	Value
Powered Electrode Voltage	150 V
Fundamental Frequency	13.56 MHz
Dielectric Tube Wall Temperature	325 K
Input Argon Gas Flow Rate	100 sccm
Outlet Pressure	0.85 Torr

The HPEM model considers the charged species of argon Ar^+ , Ar_2^+ , and the neutral species Ar , $\text{Ar}(1s_2)$, $\text{Ar}(1s_3)$, $\text{Ar}(1s_4)$, $\text{Ar}(1s_5)$, $\text{Ar}(4p)$, $\text{Ar}(4d)$ and Ar_2^* . Here, Ar_2^* is a metastable state of argon – an excited state that has a particularly long lifetime compared to most and so exists long enough for the purpose of these reactions to be considered as one of the species in neutral argon [54]. The model considers electrons in the bulk plasma in the same manner as the heavier species, but secondary electrons are considered separately via the Monte-Carlo method [39, 55].

2.6 Determining Plasma Parameters

2.6.1 DC Self Bias and Sheath Potential

The dc self bias is a crucial metric underpinning *Pocket Rocket*'s operation. The dc self bias is determined by measuring the dielectric surface plasma potential in a mesh cell axially aligned and adjacent to the powered electrode, inside the plasma (position $R = 2.1$ mm, $Z = 22$ mm, indicated by the red circle in Figure 2.1). The dc self bias is the phase-averaged value of this potential. Figure 2.3 shows the radial variation of plasma potential in the throat region at $Z = 22$ mm and the extracted values of the dc self bias and plasma bulk potential.

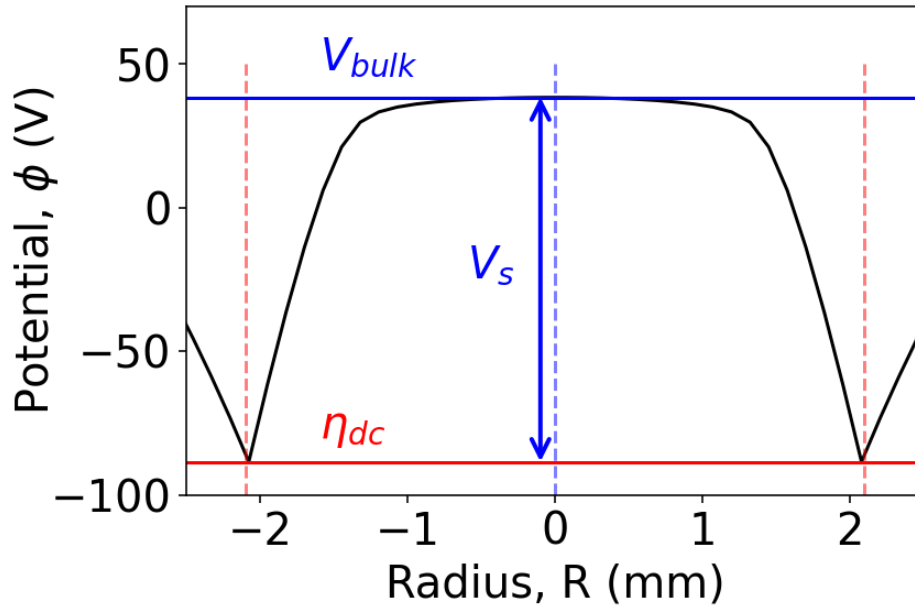


Figure 2.3: Radial variation of plasma potential at $Z = 22$ mm, axially aligned with the powered electrode. Regions at $|R| > 2.1$ mm are the dielectric material. Red dashed lines indicate the position where the electrode surface potential is measured, $R = 2.1$ mm, and the continuous red line the potential value extracted. The phase average of this value gives the dc self bias, η_{dc} . The blue line indicates the potential in the centre of the plasma bulk, phase-averaged to give V_{bulk} . The sheath potential, V_s , is found as the difference between these potentials, as in Equation 2.6.1. Data shown from simulation with peak-type waveform, $\theta_k = 90^\circ$, as an example.

The corresponding sheath potential, V_s , in each case was calculated as the difference between the potential in the centre of the bulk (i.e. far away from wall effects), V_{bulk} and the dc self bias, η_{dc} as shown in Equation 2.6.1.

$$V_s = V_{bulk} - \eta_{dc} \quad (2.6.1)$$

This method was selected, as oppose to taking the potential at the sheath extent boundary, as the difference in measured potential was small and measuring at the radial centre ensured consistency between simulations.

2.6.2 Sheath Extent

The position of the sheath edge in this work is defined using the Brinkmann criterion [56]. This models the position of the sheath edge by considering the variation of ion and electron density with position and time within the sheath. The edge is defined as where, moving from the plasma bulk outwards toward the wall, the ion and electron densities are equal either side of the sheath/pre-sheath boundary. This is depicted in Figure 2.4 and described by Equation 2.6.2.

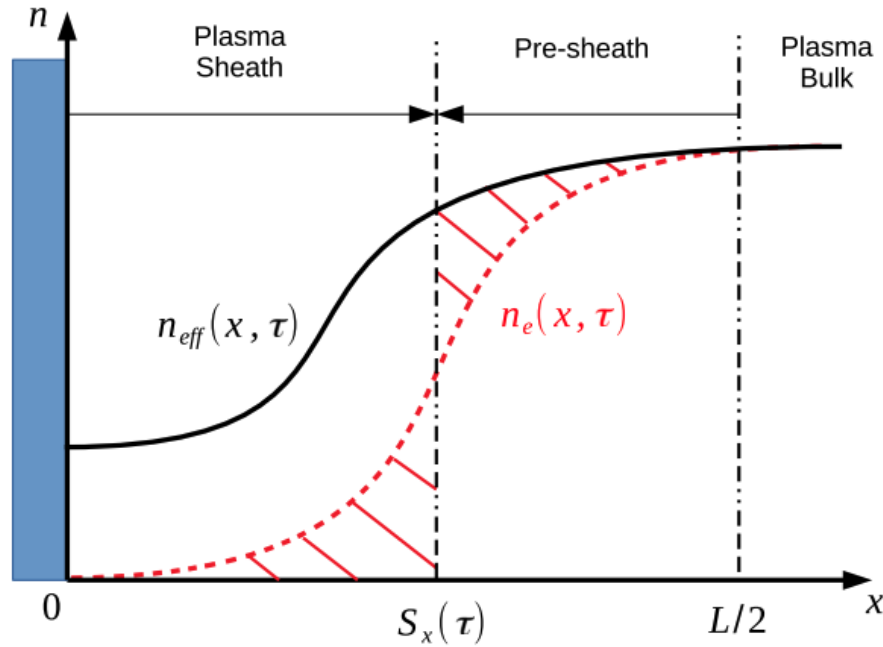


Figure 2.4: Sketch of ion and electron spatial distribution close to the wall of a plasma container at a point in time τ . Here, n_{eff} refers to the effective positive charge density due to ions and n_e the electron density. $L/2$ refers to the centre of the plasma bulk. The wall material is indicated by the blue region. At time τ the position of the sheath edge, S_x is defined by the Brinkmann Criterion. This is the position at which the red dashed areas are equal either side. This is found using Equation 2.6.2. Figure reproduced from ref. [15].

$$\int_0^{S_x(\tau)} n_e(x, \tau) dx = \int_{S_x(\tau)}^{L/2} n_{eff}(x, \tau) - n_e(x, \tau) dx \quad (2.6.2)$$

This allows visualisation of the sheath edge to compare with the spatial distribution of plasma properties in the thruster.

2.6.3 Gas Heating Power Efficiency

For the purposes of this work, the gas heating power efficiency is determined as the change in temperature generated in *Pocket Rocket* per unit power deposited, following an established method from previous work using the sinusoidal voltage waveform [40]. The temperature was measured for each peak-type waveform at a fixed position in the throat of the thruster, inside the sheath edge and axially aligned with the powered electrode (position $Z = 23$ mm, $R = 1.5$ mm). This position was chosen as it consistently measured the highest gas temperature for all waveforms. The difference between the measured temperature and the fixed wall temperature (325 K) gives the temperature increase, ΔT . The power deposition is extracted as a parameter from HPEM based on a cylindrical integration of the rf power density across the thruster volume.

2.7 Sources of Uncertainty

Reliably quantifying uncertainties in simulations is a known and substantial challenge across the low-temperature plasma physics field [57, 58]. Producing a quantitative error based on the parameters and physical assumptions underpinning HPEM is beyond the scope of this work. The sensitivity of the model to some key sources of uncertainty are presented here.

2.7.1 Temperature of Dielectric Tube Wall

For this work, the temperature of the dielectric tube walls of *Pocket Rocket* were fixed at 325 K during simulations. In Figure 2.1, this is the edge of the light grey dielectric region that meets the plasma. This does not accurately reflect what would be expected over operational timescales in practical experiment due to heat transfer from the plasma to the wall [28]. To account for this effect, previous work has established a ‘two-step’ method of establishing wall temperature: first performing the simulation at 325 K, extracting the maximum on-axis neutral gas temperature, and re-running the simulation with this temperature as the wall temperature [39]. This was not performed due to time constraints in this work.

By simulating the walls at 325 K, they act to cool the gas as it travels downstream from the point of heating (axially aligned with the powered electrode). However, in this work all gas temperature measurements were sampled at the point of heating, therefore mitigating the wall-cooling effect.

Ref. [39] compares maximum neutral gas temperature found using 325 K walls to that using the two-step method, using the sinusoidal waveform under similar operating conditions. In this case, α -mode operation resulted in a 7% neutral gas temperature increase using the ‘hot’ walls compared to cold; γ -mode operation resulted in a 33% increase. This effect is considered in final examination of gas heating power efficiency in Section 3.6.

2.7.2 Model Comparison with Fully Kinetic Treatment

Section 2.1 outlines some of the compromises hybrid modelling makes compared to fully kinetic models to reduce computational expense. To compare the methods, additional simulations were completed using HPEM to model a 1D parallel-plate CCP, as shown in Figure 1.4. This was performed at 100 mTorr and 270 V using the 13.56 MHz sinusoidal voltage waveform. This reflected the conditions used in ref. [59], modelling a 1D-CCP using a PIC simulation to enable comparison. The electron and ion densities produced using HPEM are shown in Figure 2.5.

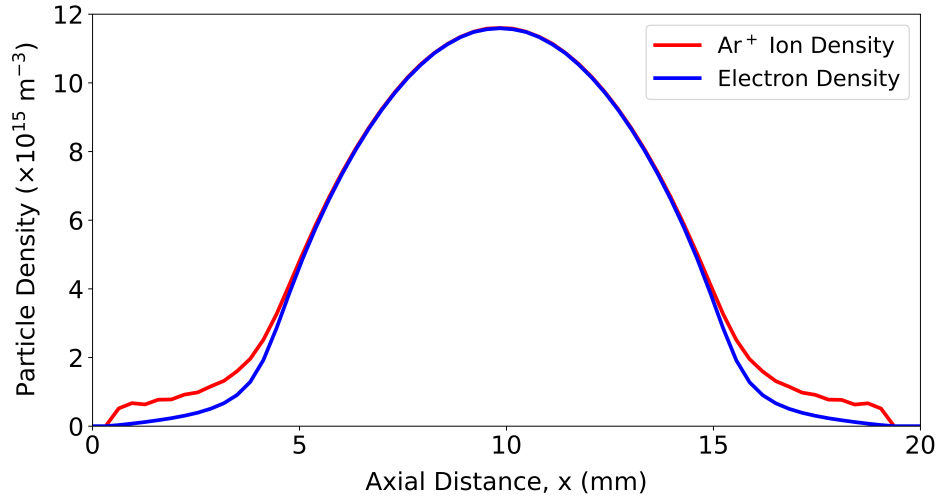


Figure 2.5: Ar⁺ and electron densities produced by simulating a 1D CCP as shown in Figure 1.4 with HPEM. The same operating conditions were employed as in ref. [59] to allow direct comparison with a particle-in-cell model.

The HPEM simulation produced the expected distribution of ion and electron densities, as introduced in Figure 1.3. Densities calculated are of the same order of magnitude as PIC simulations, although the maximum value is slightly larger when using HPEM [59]. This increase may be driven by the HPEM chemistry set considering more reaction species, such as argon metastables, compared to the PIC simulation which only considered the most fundamental plasma reactions.

Crucially, HPEM has repeatedly been qualitatively benchmarked against experimental results using *Pocket Rocket* under similar operating conditions using sinusoidal [39] and tailored waveforms [41, 42]. Therefore, this work focuses on trends observed in simulation data that will reflect practical operation of *Pocket Rocket*.

3 Results and Discussion

This chapter begins with *Pocket Rocket* operation using the 13.56 MHz sinusoidal voltage waveform, introducing key quantities and behaviours that influence the gas heating properties of the thruster. This establishes confidence in the methodology through comparison with literature results under the same conditions, benchmarked against experiment. The same characteristics are then compared from operation using the peak-type waveforms, and a description of the physical processes underpinning the different heating behaviours discussed.

3.1 *Pocket Rocket* Sinusoidal Waveform Operation

Initially, *Pocket Rocket* was simulated using a 13.56 MHz sinusoidal voltage waveform at baseline operating conditions as shown in Table 2.1. Figure 3.1 shows the voltage waveform and associated dielectric surface potential, measured as described in Section 2.6.

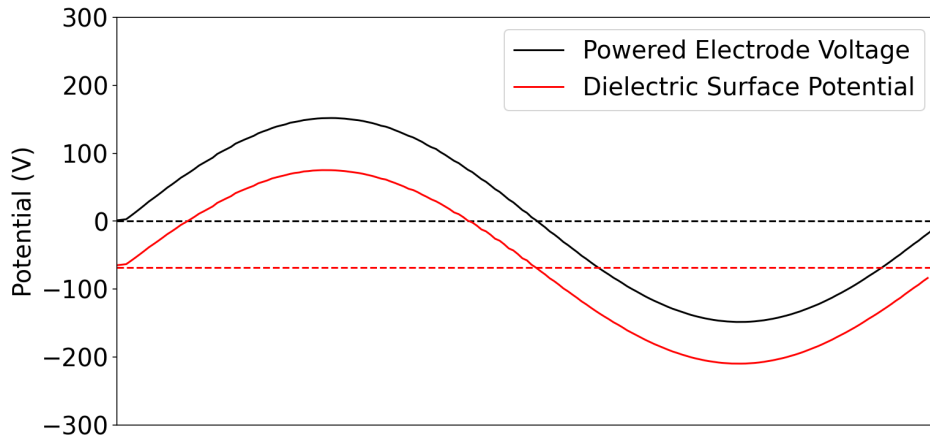


Figure 3.1: Powered electrode voltage waveform for 13.56 MHz sinusoidal operation (black). Dielectric surface potential (red), measured at dielectric surface wall axially aligned with the powered electrode ($Z = 22$ mm), as shown by red circle in Figure 2.1. The phase-averaged value is the dc self bias voltage = -67 V, (dashed red).

The dc self bias, calculated as the phase-average of the dielectric surface potential, is -67 V. This is comparable to previous work which calculated a value of -70 V [39].

The spatial distribution of power density and the axial pressure shown in Figure 3.2.

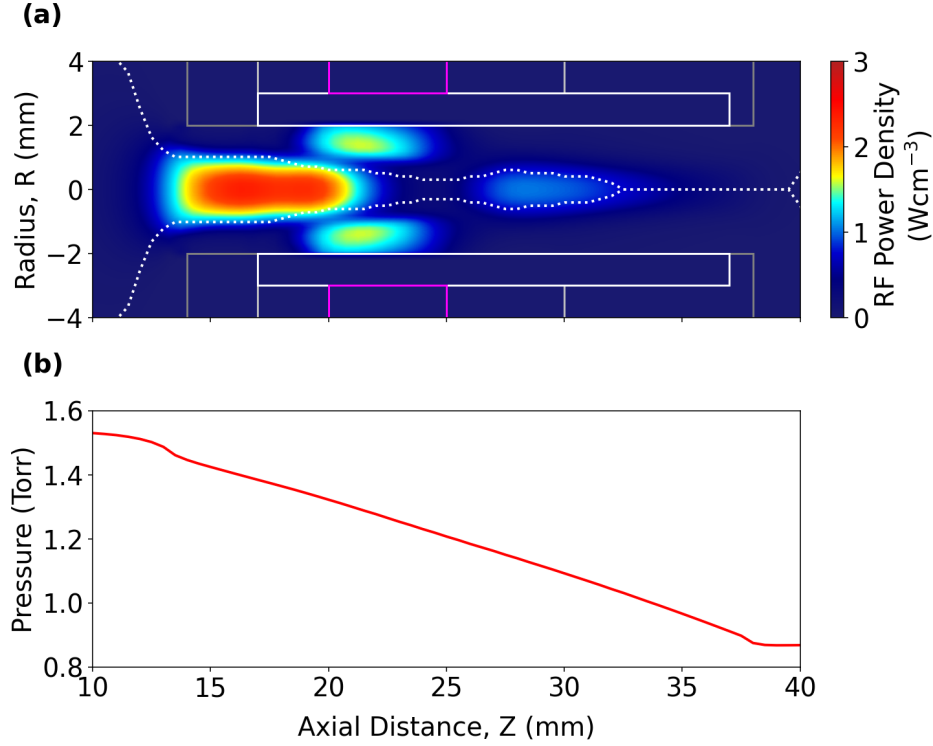


Figure 3.2: (a) Spatial distribution of power density in throat of thruster, as shown in Figure 2.1. The sheath extent is indicated by white dashed lines. Thruster is operated using 13.56 MHz sinusoidal voltage waveform and baseline operating conditions, as described in Table 2.1. (b) Axial variation of pressure across the same region.

Figure 3.2(a) shows the power density maxima occurs radially centred and upstream of the powered electrode. This is due to the pressure being higher upstream of the electrode ($Z < 20$ mm), as seen in Figure 3.2(b). Consequently, there is a greater number of particles upstream allowing for greater power deposition. A linear decrease in pressure is seen across the throat region until it reaches approximately 0.85 Torr, the outlet pressure defined in the baseline operating conditions. Both the power distribution and axial pressure profile are in qualitative agreement with previous study of the sinusoidal voltage waveform [39].

Figure 3.3 shows the spatial distribution of plasma potential, ion density, electron temperature and neutral gas temperature during sinusoidal waveform operation.

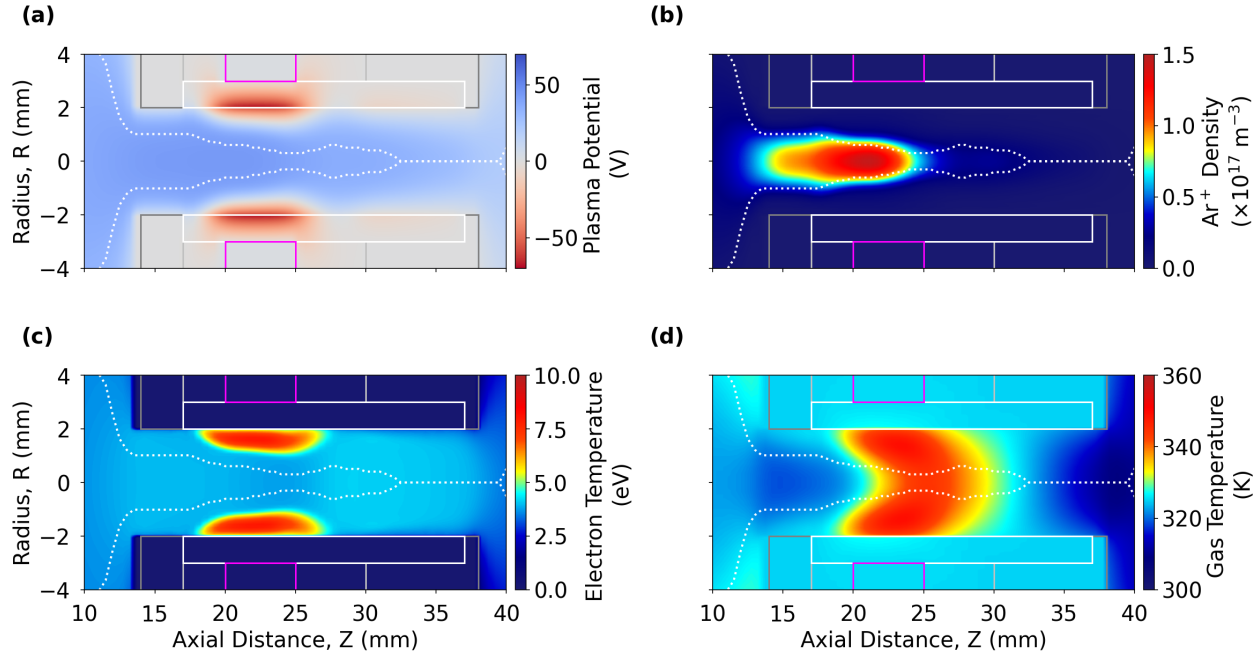


Figure 3.3: Spatial distribution of (a) plasma potential, (b) Ar^+ ion density, (c) electron temperature and (d) neutral gas temperature during 13.56 MHz sinusoidal voltage waveform operation at baseline conditions as described in Table 2.1. The sheath extent is indicated by white dashed lines in each case.

Figure 3.3(a) shows a highly negative potential forms on the surface of the dielectric, axially aligned with the powered electrode. This results in the dc self bias effect and agrees with physical theory of an asymmetrical CCP as discussed in Section 1.2.4 [13]. The resultant Ar^+ density distribution shown in Figure 3.3(b) shows the maximum ion density occurring axially aligned with the powered electrode. This is in agreement with previous work, establishing that the dc self bias drives increased ionisation axially aligned with the powered electrode [39]. Additionally, ion density decreases across the thruster due to the negative pressure gradient seen in Figure 3.2(b) [39].

Qualitatively, the electron temperature and neutral gas temperature spatial distributions in Figure 3.2(c) and (d), respectively, also agree with results in ref. [39]. However, the values obtained in this work are approximately an order of magnitude smaller. This is due to the simplified method for assessing the temperature of the wall, as described in Section 2.7.1.

Qualitative consistency with previous work has been demonstrated operating with the sinusoidal voltage waveform, establishing confidence to study the tailored waveform case.

3.2 Tailored Waveform Modulation of DC Self Bias

Figure 3.4 shows the dc self bias, as introduced in Section 2.6.1, measured using the peak-type tailored voltage waveform, described in Equation 1.2.4, with θ_k varied from 0° to 360° . Operating conditions are those described in Table 2.1, to allow comparison with sinusoidal waveform case and previous works.

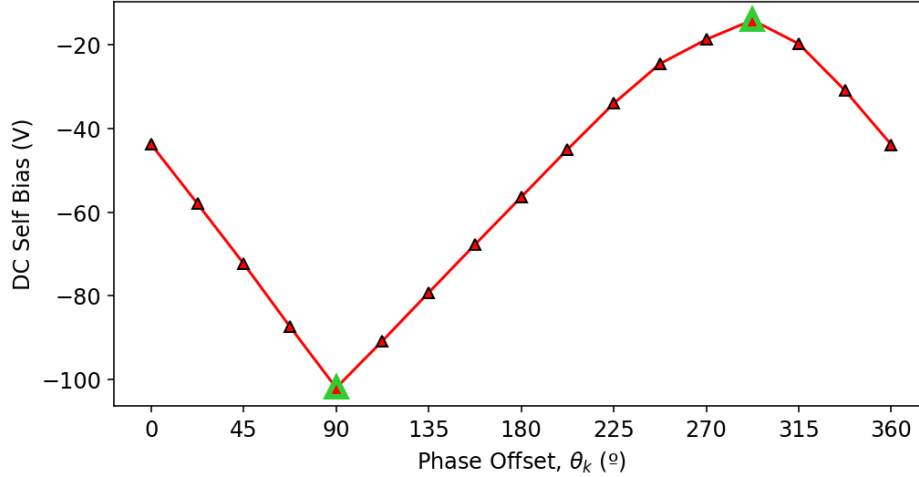


Figure 3.4: DC self bias for peak-type tailored voltage waveforms with θ_k varied from 0° to 360° . Baseline operating conditions described in Table 2.1 used. The dc self bias was modulated by 86% between extrema values, occurring at $\theta_k = 90^\circ$ and 292.5° respectively (highlighted in green).

$\theta_k = 90^\circ$ and 292.5° waveforms are identified as producing the most and least negative dc self biases, -102 V and -14 V, respectively. An 86% modulation occurs between these values, aligning with previous work using the peak-type waveform in similar conditions that produced a modulation of 85% [42].

Identifying the 90° offset and 292.5° offset waveforms as resulting in the extremes of dc self bias, the next section explores the resultant changes in plasma behaviour at a macroscopic level.

3.3 Macroscopic Plasma Parameters in DC Self Bias Extrema of Peak-Type Waveform

Establishing that the $\theta_k = 90^\circ$ and 292.5° waveforms constitute the extremes in dc self bias, Figure 3.5 compares the powered electrode voltage waveforms, the resultant dielectric surface potentials and sheath potentials.

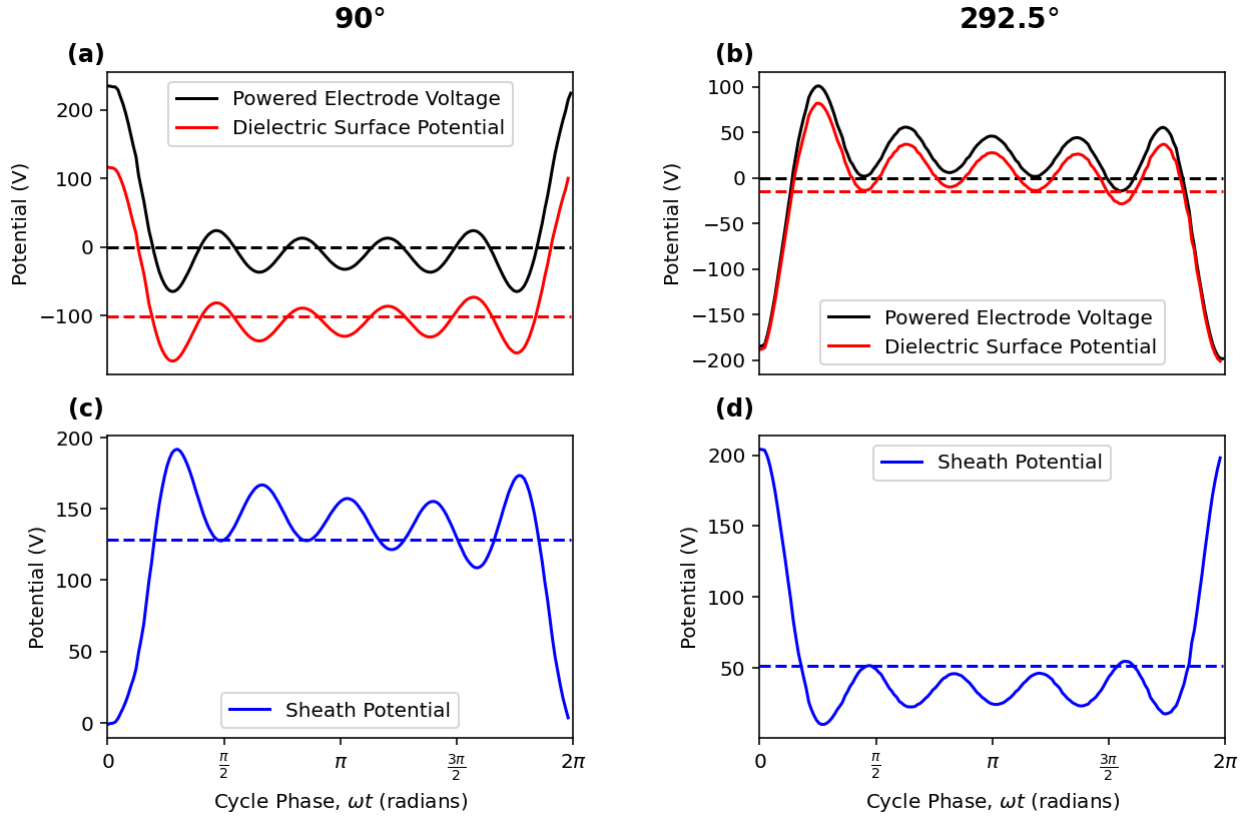


Figure 3.5: (a) and (b) show the driving voltage waveform and resultant dielectric surface potential and dc self bias for the $\theta_k = 90^\circ$ and 292.5° peak-type waveforms, respectively. The dashed red line indicates the phase-averaged dc self bias in each case. Panels (c) and (d) display the resultant phase-resolved sheath potential for each. The dashed blue line indicates the phase-averaged sheath potential, V_s , in each case. For $\theta_k = 90^\circ$, $V_s = 128$ V, factor 2.5 than to $\theta_k = 292.5^\circ$, which produced $V_s = 51$ V.

Figure 3.5(a) shows the large dc self bias generated using the $\theta_k = 90^\circ$ waveform produces a large sheath potential, V_s , as seen in Figure 3.5(c). By contrast, 3.5(b) and 3.5(d) show the small dc self bias generated by the $\theta_k = 292.5^\circ$ waveform, produced a much smaller sheath potential. Using $\theta_k = 90^\circ$ produced a sheath potential factor 2.5 larger than using $\theta_k = 292.5^\circ$. This results in greater ion acceleration, coupling more power into ions and therefore more energetic neutral-ion charge exchange collisions, cause gas heating as described in Equation 1.2.3 [13].

Figure 3.6 compares how this manifests in differences between the spatial distribution of power density, plasma potential, Ar^+ density, electron temperature and neutral gas temperature for each waveform.

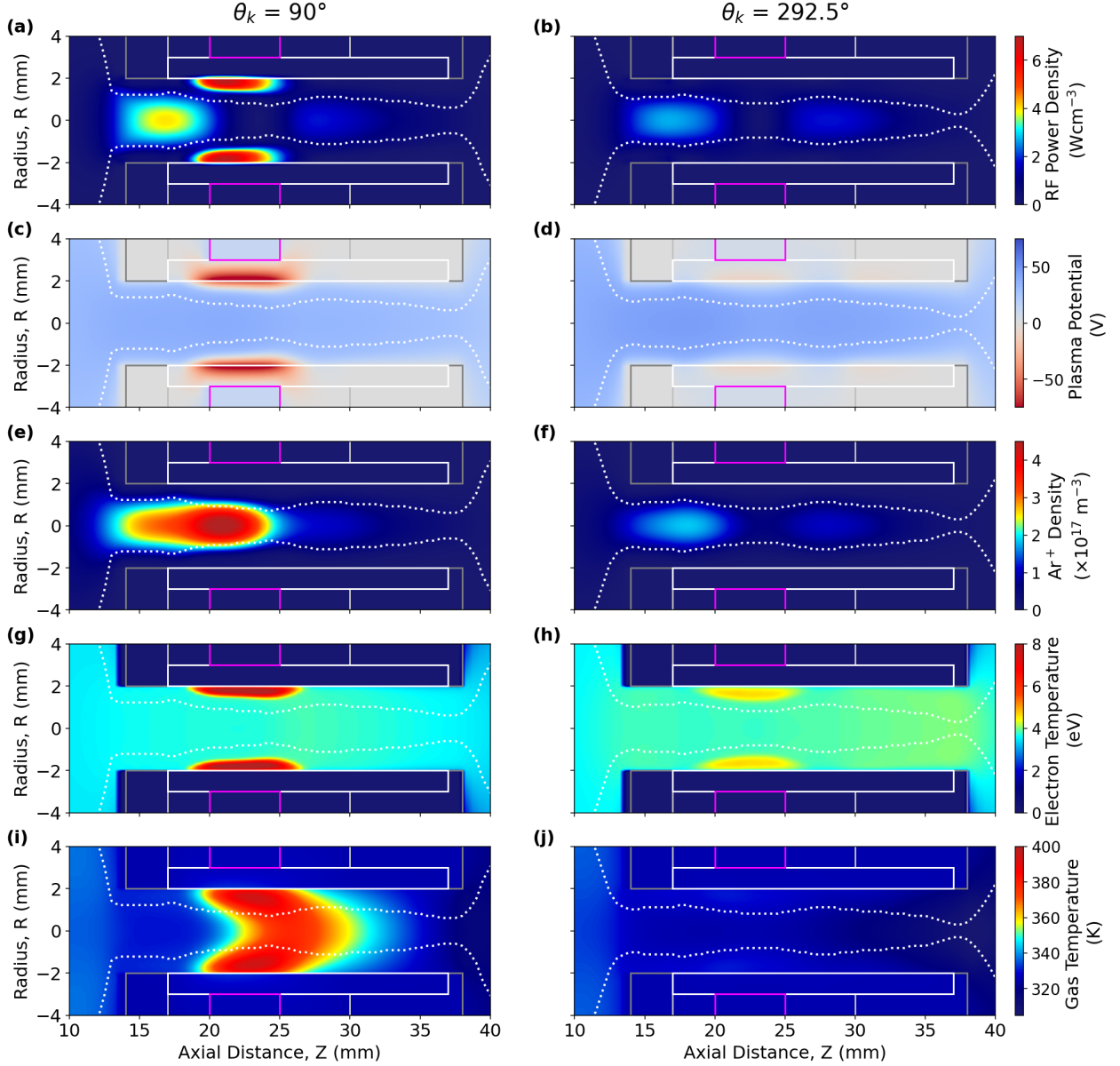


Figure 3.6: Comparison of spatial distribution of key plasma properties between $\theta_k = 90^\circ$ peak-type waveform and $\theta_k = 292.5^\circ$. Panels (a) and (b) compare the rf power density; (c) and (d) the plasma potential; (e) and (f) the Ar^+ ion density; (g) and (h) the electron temperature, (i) and (j) the neutral gas temperature. White dashed lines indicate the sheath extent in each case. Operated using baseline conditions established in Table 2.1.

Figure 3.6(a) shows the majority of the power density for $\theta_k = 90^\circ$ waveform is deposited within

the sheath and axially aligned with the powered electrode. This is consistent with established theory of γ -mode operation, where the plasma is primarily sustained through ionisation by secondary electrons, as described in Section 1.2.5 [41, 60]. By contrast, Figure 3.6(b) shows power deposition using $\theta_k = 292.5^\circ$ is very limited within the sheath. Instead, power deposited in two maxima that occur upstream and downstream of the powered electrode, with a maximum value factor 2.5 less than using the $\theta_k = 90^\circ$ waveform. Previous work under similar conditions using the peak-type waveform has shown power deposition variation of a similar magnitude [42]. This power deposition is consistent with α -mode heating [41, 60].

The resultant highly negative dielectric surface potential is seen in Figure 3.6(c) for the $\theta_k = 90^\circ$ waveform, reflected in the large dc self bias effect seen in Figure 3.5(a). By contrast, Figure 3.6(d) shows the $\theta_k = 292.5^\circ$ waveform produces a very small negative dielectric surface potential, therefore a small dc self bias, as in Figure 3.5(b).

Figure 3.6(e) shows the $\theta_k = 90^\circ$ waveform produced Ar^+ ion densities factor 2.5 larger than using $\theta_k = 292.5^\circ$. This is in agreement with previous work, observing the same factor modulation in ion density by varying θ_k [42]. The position of the density maxima in the axial centre of the dielectric tube for $\theta_k = 90^\circ$ has been observed for γ -mode operation in previous simulation and experimental works [19, 29].

Increased electron temperatures are seen in $\theta_k = 90^\circ$ operation compared to $\theta_k = 292.5^\circ$ in Figure 3.6(g) and 3.6(h), respectively. This agrees with physical theory that increased power deposition will provide more energy to electrons and therefore increase their temperature, as discussed in Section 1.2.5.

Figures 3.6(i) and 3.6(j) show neutral gas temperature increase is proportional to dc self bias and power density, agreeing with established theory using tailored waveforms [41]. Furthermore, the position of the temperature maxima is approximately invariant with changing waveform, either with varied θ_k or compared to the sinusoidal case in Figure 3.3(d). In each case, gas heating occurs inside the powered electrode sheath and diffuses toward the centre of the thruster. Regions of high power deposition upstream of the electrode in Figures 3.6(a), 3.6(b) and 3.2(a) in the sinusoidal case, are shown to have no significant contribution to neutral gas heating. Therefore, it is concluded that increased neutral gas heating is related to increased power deposition adjacent to the powered electrode driven by modulated dc self bias [39, 41]. The mechanism underpinning this effect is explored in the following sections.

3.4 Power Deposition Within the Plasma Sheath

Establishing a correlation between more negative dc self bias and increased neutral gas heating, this section explores how the changing power deposition characteristics with varied θ_k drive this phenomena.

Figure 3.7 shows the radially-resolved ion and electron power density depositions for each value of θ_k studied using the peak-type waveform.

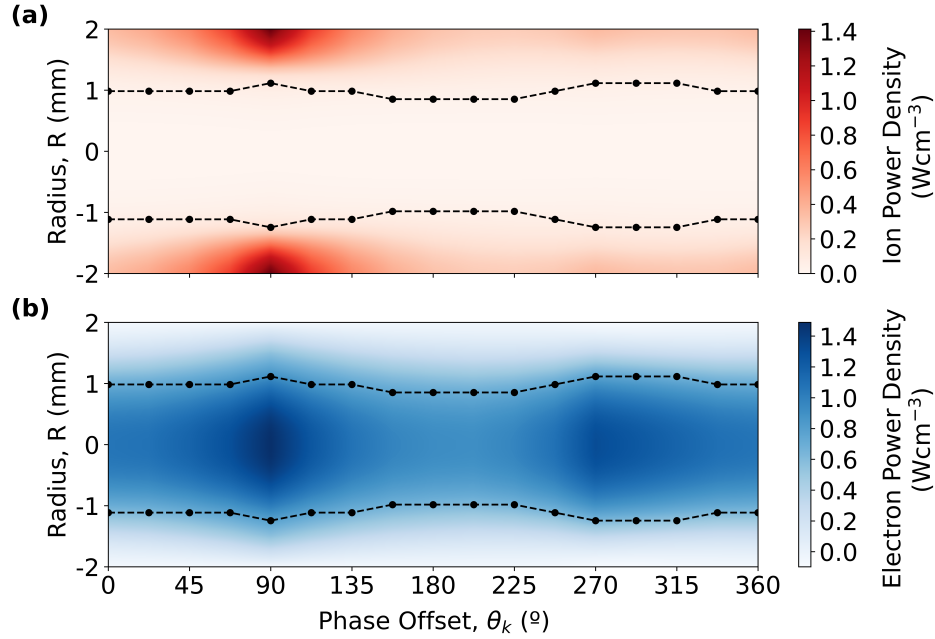


Figure 3.7: Power deposition into (a) ions and (b) electrons for peak-type waveforms with θ_k varied from 0° to 360° , axially averaged over throat region depicted in Figure 2.1. Black circles indicate the phase-averaged sheath extent for each waveform simulated and black dotted lines included to guide the eye. All simulations undertaken at baseline conditions as described in Table 2.1.

Figure 3.7(a) shows that for all phase offset values, the ion power is solely deposited inside the sheath ($|R| \gtrsim 1$ mm, black dotted line). The magnitude of deposition is proportional to the magnitude of dc self bias observed for each value of θ_k in Figure 3.4. Using $\theta_k = 90^\circ$ resulted in a factor 4.4 greater maximum ion power deposition compared to $\theta_k = 292.5^\circ$. It also correlates spatially with the position of the power density maximum shown in Figure 3.6(a). This indicates that power is coupled into ions via acceleration through the sheath potential and not via effects in the plasma bulk. This agrees with established physical theory that ions only react to time-averaged changes to the system due to their low mobility, as discussed in Section 1.2.5.

Figure 3.7(b) shows the majority of electron power for all phase offsets is deposited in the bulk region of the plasma, outside of the sheath regions. A maxima in electron power density

is observed at $\theta_k = 90^\circ$, when the ion power density is also highest. This is due to increased ionisation occurring in the bulk as a result of secondary electrons. The secondary electrons are produced by high energy ions having greater velocity and thus increasing their flux against the dielectric wall, producing secondary electrons proportional to the number incident as described in Section 1.2.5. This is further seen by increased electron power deposition close to and within the sheath at this value of θ_k . This is in agreement with previous observation of γ -mode heating using tailored waveforms [41].

3.5 Modulating Radial Ion Flux with DC Self Bias

Establishing increased ion power deposition proportional to dc self bias, Figure 3.8 shows the changing radial ion flux with θ_k for the peak-type waveform compared to the modulation of dc self bias shown in Section 3.2.

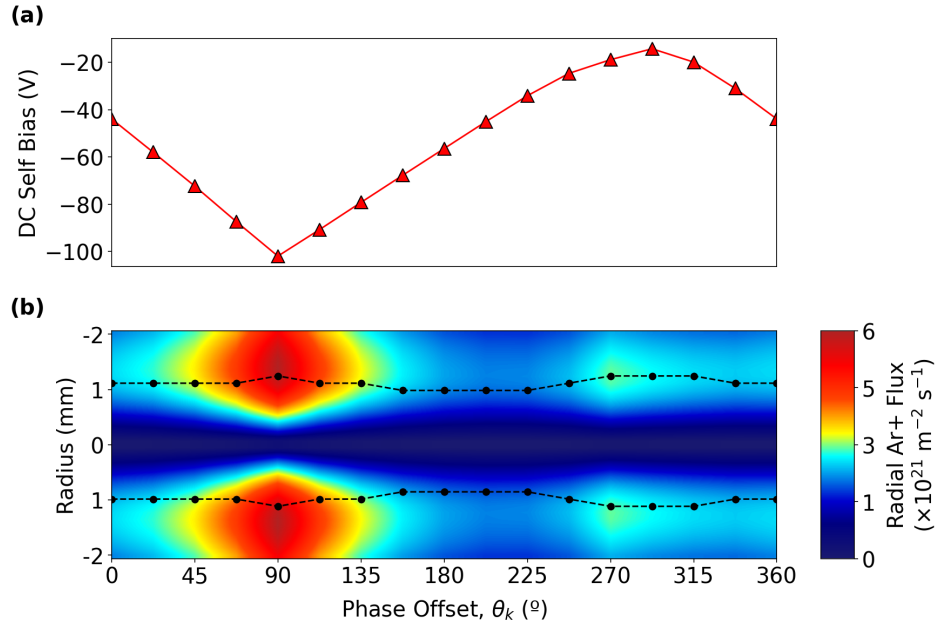


Figure 3.8: (a) dc self bias modulation due to varying θ_k in peak-type waveform, repeated from Figure 3.4 for comparison. (b) Radial Ar⁺ ion flux for varied θ_k in peak-type waveform, axially averaged over throat region shown in Figure 2.1. Operation under baseline conditions shown in Table 2.1. Largest dc self bias at $\theta_k = 90^\circ$ correlates with position of largest radial ion flux.

Figure 3.8(b) shows the maximum radial ion flux is observed at $\theta_k = 90^\circ$, agreeing with previous results correlating it with increased neutral gas heating, as shown in Figure 3.6(i). Radial ion flux increase is driven by $\theta_k = 90^\circ$ waveform producing the most negative dc self bias, as shown in Figure 3.8(a), and therefore the greatest sheath potential (Figure 3.5), accelerating more ions. Previous work has established this increased ion flux results in γ -mode heating due to the increased number of ions incident to the material wall [41, 51]. Variation with θ_k and spatial distributions of secondary electron ionisation rates are shown in Figure 3.9.

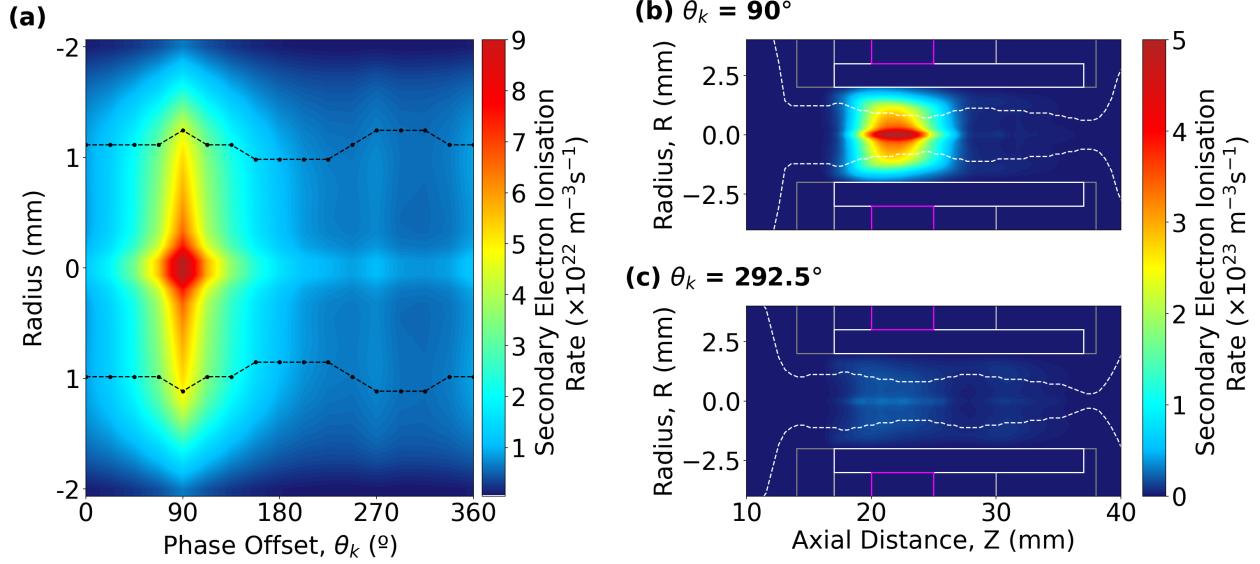


Figure 3.9: (a) Axially averaged secondary electron ionisation rate across throat region shown in Figure 2.1, with black dots indicating the sheath extent. Panels (b) and (c) give the spatial distribution in the throat region for $\theta_k = 90^\circ$ and 292.5° , respectively. All simulations using baseline operating conditions shown in Table 2.1.

Figure 3.9(a) shows that secondary electrons only make a significant contribution to ionisation rates during $\theta_k = 90^\circ$ operation, therefore it is the only case in which γ -mode heating accessed. The ionisation maxima is radially centred and axially aligned with the position of the powered electrode, as shown in Figure 3.9(b). By contrast, Figure 3.9(c) shows no significant secondary electron ionisation, as it does not access γ -mode heating. Secondary electron production drives increased ionisation in the centre of the plasma, producing more ions leading to more ion-neutral charge exchange collisions and therefore gas heating, as described in Section 1.2.6. Previous work with the peak-type waveform has observed similar increases in ion densities and fluxes due to γ -mode heating [42].

It is noted that the peak-type waveform offers a method of accessing γ -mode heating at low operating voltages, 150 V in this case. By contrast, sinusoidal waveform operation requires voltages of 300 V or more to produce γ -mode heating [39]. The peak-type waveform therefore offers a mechanism of significant modulation of neutral gas temperature electrically, enabling ‘on-the-fly’ optimisation for thruster applications.

The next section explores how varying θ_k affects the gas heating power efficiency for applications in *Pocket Rocket*.

3.6 Gas Heating Power Efficiency

While the $\theta_k = 90^\circ$ waveform has been demonstrated to generate the largest gas temperature increase, it is important to consider this effect relative to the waveform's power usage when focussing on nanosatellite applications.

Figure 3.10 shows the gas heating power efficiency, as described in Section 2.6, using the peak-type waveform with θ_k varied from 0° to 360° . For comparison, the gas heating power efficiency of the 13.56 MHz sinusoidal waveform is also shown, operated under the same conditions. Using the sinusoidal waveform, only α -mode heating is observed [39].

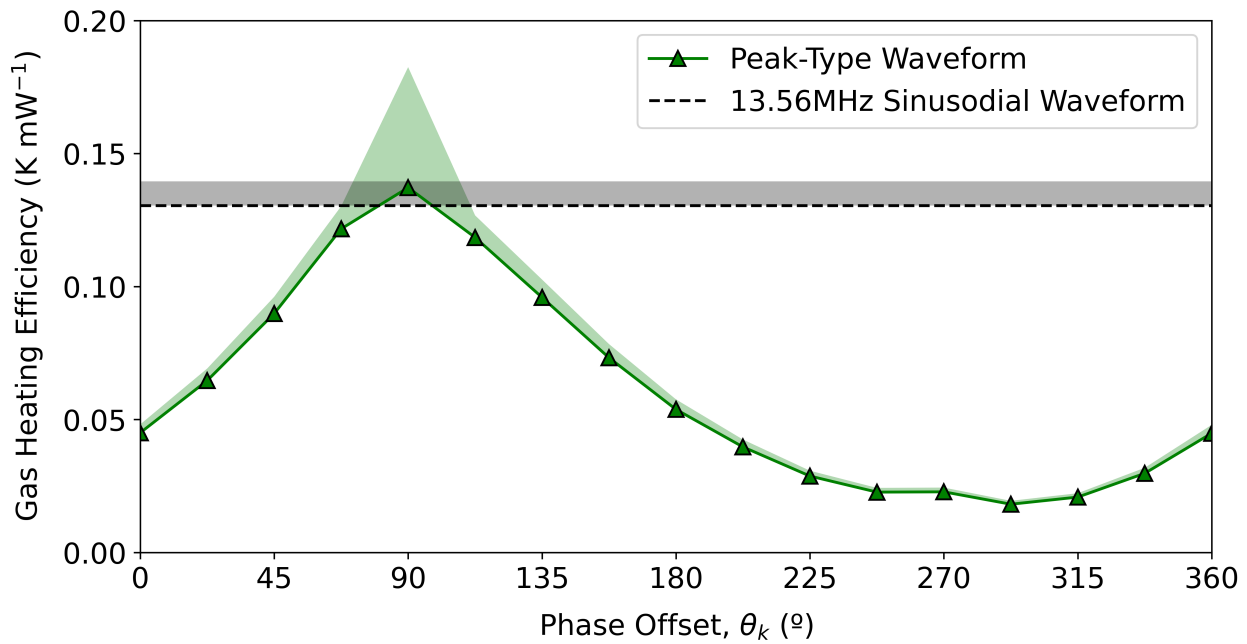


Figure 3.10: Gas heating power efficiency for peak-type waveform with θ_k varied from 0° to 360° (green), compared with 13.56 MHz sinusoidal voltage waveform (black dashed line). Values are calculated by method outlined in Section 2.6. Shaded regions in each case indicate expected values if two-step wall heating simulations undertook as described in Section 2.7.1. Simulated using baseline operating conditions described in Table 2.1.

Figure 3.10 shows using $\theta_k = 90^\circ$ results in the largest gas heating power efficiency when using the peak-type voltage waveform, a factor 7 increase compared to using the least efficient waveform, $\theta_k = 292.5^\circ$. This agrees with previous work establishing a non-linear increase in ion density with power deposition when γ -mode is accessed [42], driving increased gas heating via ion-neutral collisions without significant power draw increase [26].

Furthermore, as using $\theta_k = 90^\circ$ is the only peak-type waveform that accesses γ -mode, it is the only waveform to produce a greater gas heating power efficiency than the sinusoidal waveform, by 5%. This only applies to 150 V operation employed in this work: previous work with the

peak-type waveform has shown much larger secondary electron ionisation rates are produced for a range of θ_k values at higher electrode voltages [42]. Therefore, further investigation is required to see if other values of θ_k can result in increased efficiency compared to the sinusoidal case at different voltages.

The gas heating power efficiency values produced in this work are of the same order of magnitude as those produced in previous work studying the sinusoidal waveform [40]. However, they will be underestimates of true values due to the two-step wall heating method not being considered, as described in Section 2.7.1. The shaded regions indicate the expected increase efficiency expected by applying the two-step method, extrapolated from comparison of the method in the sinusoidal case at 150 V in [39]. Here, it is suggested that using $\theta_k = 90^\circ$ in the peak-type waveform could result in gas heating power efficiency up to 36% larger than the sinusoidal waveform.

3.7 Considerations for Future Work

Implementing the peak-type waveform for use in *Pocket Rocket* would require developing a much more complex signal generator, with potential penalties in volume and weight. By contrast, the 13.56 MHz sinusoidal waveform is the industry standard and can be generated using simple, well established power sources. Further investigation is required using a two-step wall heating method to validate the predictions made in Figure 3.10 to assess the heating efficiency gains against increased complexity implementing the peak-type waveform.

Previous work has demonstrated that varying the fundamental frequency using the sinusoidal voltage waveform produces significant changes in gas temperature [40]. Increasing the frequency above 13.56 MHz resulted in larger temperature increases but at the cost of drawing more power. Waveforms with frequency below 13.56 MHz were found to be the most efficient at gas heating. This was a result of γ -mode heating dominating at low frequencies. Varying the fundamental frequency used in the peak-type waveform could induce similar effects, broadening the values of θ_k for which γ -mode heating occurs or further enhancing the efficiency of the $\theta_k = 90^\circ$ waveform.

This work has focussed on the use of the ‘peak-type’ waveform, but other tailored waveforms have been experimentally benchmarked using *Pocket Rocket*, such as the sawtooth waveform. This produces a single, large change in voltage over an rf-cycle compared to the peak-type waveform producing two regions of comparatively smaller change. The sawtooth waveform has also been demonstrated to modulate the dc self bias by varying θ_k , and produce change in radial ion flux [15]. A similar investigation to this study could assess the gas heating properties of the sawtooth waveform and investigate how its more extreme shape effects the gas heating properties of *Pocket Rocket*.

4 Conclusion

The effect of tailored voltage waveforms on neutral gas heating mechanisms in *Pocket Rocket* microthruster was investigated using 2D fluid/Monte-Carlo simulations with the Hybrid Plasma Equipment Model. Operating conditions were kept standard at 150 V electrode potential, 325 K wall temperature, 100 sccm argon propellant flow and 0.85 Torr outlet pressure, with variation only in the phase-offset, θ_k , of the peak-type tailored waveform. This resulted in 86% modulation of the dc self bias. Using $\theta_k = 90^\circ$ produced the most negative dc self bias, resulting in the largest sheath potential. This results in the largest ion acceleration and therefore the greatest ion power deposition. Ion-neutral collisions are therefore more energetic resulting in increased gas heating. Additionally, increased ion acceleration results in increased radial ion flux to the dielectric wall; using $\theta_k = 90^\circ$, this is sufficient to release secondary electrons from the surface, enabling γ -mode heating. Significantly, the peak-type tailored waveform offers a mechanism of accessing γ -mode at approximately half the operating voltage required using the established 13.56 MHz sinusoidal waveform. It therefore provides a novel mechanism to control the gas heating of *Pocket Rocket* entirely electrically, for use in remote optimisation of thruster performance once it has been deployed. Using $\theta_k = 90^\circ$ produces the greatest gas heating power efficiency using the peak-type waveform, factor 7 greater than the least efficient phase offset value, $\theta_k = 292.5^\circ$. However, $\theta_k = 90^\circ$ produced only a 5% increase in power efficiency compared to using the sinusoidal waveform. Further investigation using the two-step wall heating method is required to assess power efficiency gains quantitatively.

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